Student understanding of the wave nature of matter: Diffraction and interference of particles

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This paper reports on a study of student understanding of the wave nature of matter in the context of the pattern produced by the diffraction and interference of particles. Students in first-year, second-year, and third-year physics courses were asked to predict and explain how a single change in an experimental setup would affect the pattern produced when electrons or other particles are incident on a single slit, double slit, or crystal lattice. The errors made by students after standard instruction indicated the presence of similar conceptual and reasoning difficulties at all levels. Among the most serious was an inability to interpret diffraction and interference in terms of a basic wave model. Other errors revealed a lack of a functional understanding of the de Broglie wavelength. Students often treated it as a fixed property of a particle, not as a function of the momentum. An important goal of this investigation was to provide a research base for the design of instruction to help students develop and apply a basic wave model for matter.

I. INTRODUCTION

In this paper, we describe an investigation of student understanding of the wave nature of matter in the context of the pattern produced by the diffraction and interference of particles. The purpose was to identify and analyze the difficulties encountered by university students in introductory and more advanced courses in trying to account for these phenomena. Earlier studies by our group had demonstrated that traditional instruction typically does not result in the development of a coherent conceptual framework for geometrical and physical optics.[1–5] To address this problem, we produced research-based tutorials that have proved effective in helping students construct and apply ray and wave models for light.[6,7] Therefore, an important additional motivation for the present investigation was the design of a tutorial to help students apply a phenomenological wave model for matter.[8]

In courses on modern physics, an analogy to physical optics is typically used to introduce the idea that electrons and other particles can behave like waves. Interference patterns produced by light and by electrons provide the basis for a discussion of the wave–particle duality. The de Broglie wavelength is defined and used in explaining electron diffraction, the Davisson–Germer experiment, and other experiments in which the wave-like properties of matter are relevant. Often the de Broglie wavelength serves as a bridge to the formal study of quantum physics. It expresses the inseparable linkage of the momentum of a particle to a spatial property of the particle. As such, the de Broglie wavelength can form the basis for determining, in a given system, whether to apply classical or quantum mechanics. It is invoked by many instructors as a means of accounting for the qualitative shape of wave functions, particularly those of stationary states that can be described semiclassically (e.g., using the WKB approximation).[9] In view of the central role of the de Broglie wavelength in introductory quantum mechanics, we were especially interested in examining the ability of university students to interpret and apply this concept in the first context in which it is usually introduced—the diffraction and interference of particles. There has been relatively little research on student understanding of this particular topic. Previous studies on the wave nature of matter have focused on the Bohr atom, energy levels, and wave–particle duality and have mostly involved precollege students.[10]

II. OVERVIEW OF RESEARCH

There is a growing research base that can guide the development of curriculum in physics, especially at the introductory level.[11] The Physics Education Group at the University of Washington takes an empirical approach. Typically, we investigate student understanding through a combination of individual demonstration interviews and the administration of written problems. The results are used to guide the development and assessment of curriculum to improve student learning.

The present study involved students who were enrolled in courses from introductory to more advanced levels.[12] All had received standard lecture instruction on the diffraction and interference of electrons and other particles in their current or previous courses. The de Broglie wavelength had been explicitly covered.

Written problems on the diffraction and interference of electrons were administered to more than 450 students enrolled in a variety of physics courses: an algebra-based class, two calculus-based classes, three second-year modern physics classes, and three third-year quantum mechanics classes. Most of the problems were on course examinations, but some were used as pretests for the tutorial described later in this paper. One of the problems was also given to a calculus-based class at another university. The results were similar to those obtained at the University of Washington.

The interviews were conducted with 14 students from a third-year quantum mechanics course. They were all volunteers whose final grades were at or above the class mean. The interview transcripts yielded insights that helped us design the written problems and interpret some of the student responses.

There were several different versions of the written problems. Some involved single-slit diffraction of electrons; some involved double-slit interference of electrons; and others were on electron diffraction from a crystal lattice. [See

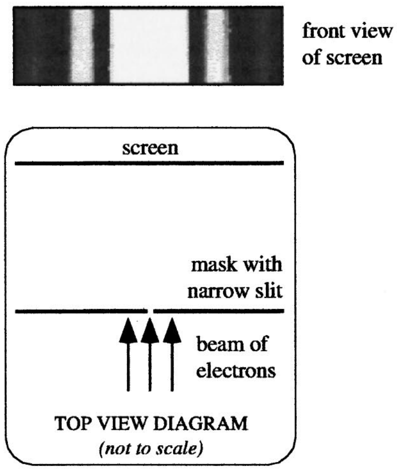
A beam of mono energetic electrons is incident on a mask containing a single narrow slit.

A. For each change below, would that change cause the minima to move closer together, move farther apart, or stay at the same locations? *Explain your reasoning.*

i. The slit width is halved.

ii. The kinetic energy of the electrons is halved.

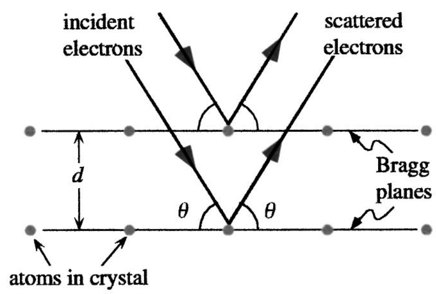
B. Suppose that the electrons were replaced with particles of greater mass such that the resultant pattern was **exactly the same** as that in the original experiment. In this case, would the kinetic energy of the new particles be greater than, less than, or equal to that of the original electrons? *Explain your reasoning.*



(a)

Mono energetic electrons are incident on a crystal lattice. Intense scattering is observed at angles θ according to the Bragg condition, $2d \sin\theta = n\lambda$.

For each change below, determine whether that change would cause the angles θ for intense scattering to become larger, smaller, or stay the same. *Explain your reasoning in each case.*



i. The target is replaced with another crystal that has the same lattice structure but a smaller lattice spacing. (Consider the Bragg planes analogous to those indicated above.)

ii. The speed of the incident electrons is decreased.

ii. The electrons are replaced with neutrons, with each neutron having the same kinetic energy as each of the original electrons.

(b)

Fig. 1. Examples of written problems posed on examinations and pretests. Students were asked to predict the effect on the positions of the maxima (or minima) of making specified changes in the experimental setup when electrons or other particles are incident on a single slit, double slit, or crystal lattice. Each problem included two types of questions. The first (type S) probed student understanding of a basic wave model and the second (type P) probed student understanding of the de Broglie wavelength.

Figs. 1(a) and 1(b) for typical examples.] Two types of questions were asked: type S and type P. The first involved changes to the slit width, slit separation, or lattice spacing. The type P questions involved changes to the momentum. Each problem began with one question of type S and included one or more questions of type P. In each question, the students were asked to predict and explain how a single change in the experimental setup would affect the positions of the maxima (or minima) of the interference (or diffraction) pattern.

On questions of type S, students were asked about the effect on the spacing of the interference or diffraction fringes (maxima or minima) when the slit width *a*, slit separation *d*, or lattice spacing is varied. In the introductory and modern physics classes and one of the quantum mechanics classes, the type S question was based on a photograph of a pattern produced on a distant screen by electrons incident on a single (or double) slit. In the other quantum mechanics classes, the question was based on the Davisson–Germer experiment. The students were shown the diagram in Fig. 1(b) and given the condition for Bragg scattering ($2d \sin\theta = n\lambda$). They were asked to predict the effect of using a crystal with a smaller lattice spacing while considering the same Bragg planes.

All type S questions could be answered solely on the basis of an analogy to a wave model for light. Acceptable responses included explanations based on changes in path length (or phase) or reference to the equations $a \sin\theta = m\lambda$ (or $d \sin\theta = m\lambda$). In the question based on the Davisson–Germer experiment, the decrease in lattice spacing would result in a decrease in the path length difference at a given angle between beams from adjacent Bragg planes. Therefore, there would be an increase in the angles (θ) for constructive interference.

Questions of type P probed student understanding of the factors affecting the de Broglie wavelength of the incident particles. The students were told that the incident electrons were monoenergetic or that they had been accelerated from rest through a potential difference V_0. The students were asked: (a) to predict the effect on the spacing of the fringes

Table I. Results from type S questions on diffraction or interference of electrons in which students were asked to predict the effect on the positions of the maxima (or minima) of changing the slit width, slit separation, or crystal lattice spacing. Questions were administered: (a) post-lecture instruction and (b) post-tutorial instruction. [Percentages have been rounded to the nearest 5%.]

Type S Questions	Algebra-based course %	(N)	Modern physics course %	(N)	Quantum mechanics course %	(N)	Calculus-based course[a] %	(N)
(a) **Post-lecture instruction**	(N=103)		(N=100)		(N=95)		(N=169)	
Correct responses	**35%**	**(36)**	**60%**	**(60)**	**65%**	**(61)**	**85%**	**(144)**
with correct reasoning	5%	(5)	25%	(23)	35%	(32)	45%	(73)
(b) **Post-tutorial instruction**	(N=103)		(N=18)		(N=52)		(N=96)	
Correct responses	**60%**	**(62)**	**85%**	**(15)**	**75%**	**(40)**	**95%**	**(91)**
with correct reasoning	40%	(42)	65%	(10)	45%	(24)	90%	(86)

[a]Students in the calculus-based classes had previously completed a series of tutorials on physical optics.

of varying the accelerating voltage, speed, or kinetic energy of the electrons; (b) to predict the effect on the spacing of the fringes of replacing the electrons with particles of different mass but with the same kinetic energy as the electrons; or (c) to compare the kinetic energy of the electrons to that of particles of different mass that produce the same fringe spacing.

In neither type S nor type P questions were the terms ''wavelength'' or ''de Broglie wavelength'' mentioned. On both types of questions, multistep reasoning is required since it is necessary to determine whether the pattern would change and, if so, to predict qualitatively the new locations of the interference (or diffraction) maxima (or minima).

III. RESULTS FROM WRITTEN PROBLEMS AND INTERVIEWS

Below, we discuss the performance of students on type S and type P questions and identify some specific difficulties. Several different versions of each type were given. When different versions were used in several classes at the same level, the results were similar. Therefore, the different versions have been treated as equivalent and the results at each level have been combined. All questions were given after standard instruction.

A. Questions on changes in the slit width, slit separation, or lattice spacing (type S)

Table I(a) contains results from written responses to questions of type S, in which the slit width, slit separation, or lattice spacing is varied. The percentages given to the tables in this paper have been rounded to the nearest 5%. This represents our best estimate of the generalizability of the data among equivalent populations.

1. Performance of students

The first three columns in Table I(a) indicate that about 35% of the students in the algebra-based course and about 60%–65% of the modern physics and quantum mechanics students gave correct answers for electrons. The corresponding percentages of students in these courses who gave correct explanations ranged from about 5% in the algebra-based class to about 25%–35% in the other classes. As the fourth column in Table I(a) shows, about 85% of the students in the calculus-based course gave correct answers, with about 45% giving correct reasoning. In Sec. III A 3, we discuss how the background of these students differed from that of the students in the other courses.

2. Identification of specific difficulties

Most of the errors on type S questions reflected difficulties similar to those that we had identified in the context of light.[4,7] Many students who gave correct answers seemed to be aware that the pattern depends on the wavelength but did not give correct explanations. Some of these students based their arguments on the formulas for two-source interference or single-slit diffraction, but often did not apply these correctly (e.g., used the formula for interference maxima to refer to diffraction minima).

Whether or not they gave a correct answer, students who did not refer to the formulas rarely argued on the basis of *differences* in path length. In the algebra-based course, about 15% explicitly stated that the interference or diffraction fringes are more closely spaced when the *paths* from the slit (or slits) to the screen lie closer together (e.g., when the slit spacing or slit width is decreased). Many seemed to think of the maxima (or minima) as arising from interactions taking place along the entire path to the screen.

> ''Bright regions would move closer together because there will be more chance of overlap between the crests.'' [algebra-based course]

> ''The bright regions would get closer because the electrons are confined to less space.'' [algebra-based course]

In the quantum mechanics classes in which the context of the written question was electron diffraction from a crystal lattice, about 25% of the students did not use the given equation for Bragg scattering or refer to differences in path length or phase. Some of these students correctly stated that a smaller lattice spacing would bring adjacent beams of scattered electrons closer together, but indicated that this change would not affect the angles at which the most intense scattering would occur. (All quotes are from written responses.)

Table II. Results from type P questions on diffraction or interference of electrons in which students were asked to predict the effect on the positions of the maxima (or minima) of changing the speed or kinetic energy of the particles. Questions were administered: (a) post-lecture instruction and (b) post-tutorial instruction. Questions about the effect of changing the velocity or kinetic energy were not given in all of the classes. [Percentages have been rounded to the nearest 5%.]

Type P Questions	Algebra-based course		Modern physics/ quantum mechanics courses		Calculus-based course[a]	
	%	(*N*)	%	(*N*)	%	(*N*)
(a) **Post-lecture instruction**	(*N*=103)		(*N*=152)		(*N*=169)	
Correct responses	**20%**	**(21)**	**40%**	**(64)**	**30%**	**(48)**
With correct reasoning	10%	(10)	30%	(47)	10%	(15)
(b) **Post-tutorial instruction**			(*N*=43)		(*N*=96)	
Correct responses	⋯		**70%**	(30)	**75%**	**(74)**
with correct reasoning	⋯		65%	(28)	65%	(63)

[a]Students in the calculus-based classes had previously completed a series of tutorials on physical optics.

> ''θ shouldn't change → the lattice structure hasn't changed any: just the possible atoms to scatter off of.'' [quantum mechanics course]

> ''[The angle to the first minimum is] smaller—the incident electrons will 'fit' through the layers at smaller angles....'' [quantum mechanics course]

3. Commentary on performance of students in calculus-based physics

Typically, we would expect students in the calculus-based course to perform at about the same level as students in the algebra-based course and not as well as students in more advanced courses. (It has been our experience that results on many conceptual questions are similar in the calculus-based and algebra-based courses.[13]) However, the calculus-based classes did significantly better than the algebra-based class and also apparently better than the modern physics and quantum mechanics classes. [See Table I(a).]

In an effort to account for the high success rate of students in the calculus-based class, we examined the performance of students in other calculus-based physics classes who had responded to a similar question on the diffraction and interference of light. About 95% of the students gave a correct answer for light, with 65% reasoning correctly. The calculus-based course at the University of Washington includes a series of tutorials on the diffraction and interference of light.[14] The students in this course who were given the electron diffraction question had recently worked through these tutorials. We have found that student performance does not vary much in sections of the same course in the same or different academic quarters. Therefore, we attribute the better performance of the calculus-based physics class on the question involving electrons to the tutorials they had worked through earlier in the academic quarter.[15]

B. Questions on factors affecting the de Broglie wavelength of the incident particles (type P)

Many students who gave correct answers to type S questions did not do so for type P questions. They did not seem to realize that changes in the speed or type of particle could affect the momentum, which would affect the de Broglie wavelength. This chain of reasoning is necessary to determine whether the pattern would change. A similar logical chain is required to compare the kinetic energies of particles of different mass that produce the same fringe spacing.

1. Performance of students

In Table II(a) is a summary of the results on type P questions in which students were asked to predict the effect on the pattern of varying the accelerating voltage, speed, or kinetic energy of the electrons. In Table III(a) are results from the type P questions in which the electrons are replaced with particles of different mass. The errors made by students at all levels of instruction were similar in nature. There was some variation, however, in the frequency of particular errors among the different classes. For example, as can be seen in Table II(a), the percentage of students who gave correct answers and used correct reasoning to relate the de Broglie wavelength to the speed or kinetic energy varied from about 10% in the algebra-based and calculus-based courses to about 30% in the modern physics and quantum mechanics courses.[16]

The students in the calculus-based course performed at about the same level as the students in the algebra-based course on the type P questions. Although the students in the calculus-based course had worked through the tutorial series on physical optics, that experience does not seem to have had much effect on their performance on the type P questions. They had, however, performed much better on the type S questions, which involve changes to the slit width, slit separation, or lattice spacing. The primary difficulties that these students (and the students in the other courses) had in answering the type P questions indicated a failure to relate the de Broglie wavelength to the mass and momentum (or velocity) of a particle.

2. Identification of specific difficulties

Analysis of the reasoning given by the students enabled us to identify some common difficulties with the de Broglie wavelength. We have organized these into three broad, overlapping categories: (a) failure to recognize the relevance of the de Broglie wavelength to the interference or diffraction pattern of particles, (b) failure to relate the de Broglie wavelength to the momentum of particles, and (c) failure to treat particles with and without mass differently (in the nonrelativistic limit).

Table III. Results from type P questions on diffraction or interference of electrons in which students were asked to predict the effect on the positions of the maxima (or minima) of replacing electrons with other particles. Questions were administered: (a) post-lecture instruction and (b) post-tutorial instruction. [Percentages have been rounded to the nearest 5%.]

Type P Questions	Algebra-based course %	(*N*)	Modern physics/ quantum mechanics courses %	(*N*)	Calculus-based course[a] %	(*N*)
(1) Post-lecture instruction	($N=103$)		($N=195$)		($N=169$)	
Correct responses	**20%**	**(21)**	**35%**	**(68)**	**20%**	**(36)**
with correct reasoning	5%	(5)	25%	(46)	10%	(14)
(2) Post-tutorial instruction	($N=103$)[b]		($N=45$)		($N=96$)	
Correct responses	**85%**	**(85)**	**65%**	**(29)**	**65%**	**(63)**
with correct reasoning	65%	(66)	55%	(25)	55%	(53)

[a]Students in the calculus-based classes had previously completed a series of tutorials on physical optics.
[b]Evidence from other research (discussed in the paper) suggests that the version of the question given post-lecture and post-tutorial in the algebra-based course was easier than that asked in the other courses.

The primary focus of this investigation was on student understanding of the wave nature of matter. The questions that the students were asked referred only to the locations of the fringes. Some students, however, spontaneously stated that there would be a change in the intensity of the pattern or in the rate at which it was formed. Student understanding of these other aspects of the pattern was not probed.

a. Failure to recognize the relevance of the de Broglie wavelength to the interference or diffraction pattern. Many students failed to recognize that a change in the speed or mass of the incident particles would affect the positions of the interference (or diffraction) maxima (or minima). They often claimed explicitly that the locations of the fringes would not change. Some of these students had recognized on the preceding type S question that the pattern depends on the wavelength of the incident particles. However, these students made no reference to the de Broglie wavelength in their answers to the type P questions about whether certain changes involving the particles would affect the pattern.

(1) Questions on varying the accelerating voltage, speed, or kinetic energy of the electrons. Many students treated the spacing of the maxima (or minima) as independent of the speed of the incident particles. They seemed to think of the pattern as only a function of the slit spacing (or width) and independent of the motion of the particles. They did not, on their own initiative, relate changes in the velocity to changes in the momentum and thus to the wavelength.

> ''[Increasing the speed] will not change the location of the bright regions since the slits which cause the refraction [sic] do not change.'' [algebra-based course]

> ''[If the speed is decreased, then the minima] would stay in the same location. The location of the minima is not dependent on the speed of the electrons, if anything the screen would be dimmer overall.'' [calculus-based course]

(2) Questions on replacing the electrons with different particles. Failure to relate the pattern to the de Broglie wavelength was especially apparent when students tried to predict the effect of replacing electrons with heavier particles of the same kinetic energy. Many students stated explicitly that the change would affect the velocity of the particles; yet they decided that the pattern would not change. They failed to recognize that the momentum would decrease and thus that the de Broglie wavelength and the spacing of fringes in the pattern would change.

> ''So, if neutron replaces electron, the $v\downarrow$ [decreases]. Thus the bright region will become dimmer while showing the same pattern.'' [algebra-based course]

> ''Since the neutrons' masses are so much greater than those of the electrons, the neutrons would have to be moving much slower than the electrons were initially. Since the neutrons would be moving to the order of 10^3 times slower, again there would be fewer particles striking the screen each second... . The pattern would remain in the same form as before (picture shown above, except a lot dimmer).'' [modern physics course]

> ''[The only difference] is the rate muons go. ∴ No change in intensity, just takes longer to make pattern.'' [quantum mechanics course]

b. Failure to relate the de Broglie wavelength to the momentum. Some students recognized that the de Broglie wavelength was relevant but treated it as a fixed property of the particle. They predicted that the maxima (or minima) would remain in the same locations as before under the changes proposed in the type P questions. These students did not realize that the de Broglie wavelength is a function of the momentum (and hence the velocity) of a particle. This type of reasoning was common in all of the classes.

> ''It doesn't matter what the speed of the electrons is, the wavelength stays the same and the same pattern is produced—no change.'' [algebra-based course]

> ''[The diffraction minima] stay at the same location. Even at lower speeds, electrons still exhibit wave-like motion. As long as its wavelength stays the same, the pattern should stay the same.'' [calculus-based course]

> ''[The diffraction minima] stay at the same locations: I don't think that slowing the electrons down would change anything. We've been comparing electrons to light. We never took velocity into account when dealing with light, so the same should hold here.'' [calculus-based course]

Students often used equations such as $d \sin \theta = m\lambda$ or $a \sin \theta = m\lambda$ in order to determine how a change in slit separation d or slit width a would affect the locations of the interference or diffraction fringes. However, many failed to interpret ''λ'' as a quantity that depended on the mass or speed of the particles. Below are two examples.

> [After writing and using the equation $a \sin \theta = m\lambda$ for minima from a single slit of width a] ''Speed is not an influential factor in this event. If it were, v would be included in the single slit equations.'' [algebra-based course]

> ''Nothing because [the pattern] is independent from the speed. It will stay the same. $a \sin \theta = m\lambda$.'' [calculus-based course]

c. Failure to treat particles with and without mass differently. Students in all of the classes used inappropriate equations to relate the wavelength of the electrons (or other particles) to velocity or kinetic energy. In particular, many applied certain relationships that are valid for light (and for other types of waves) but not for matter. Rote use of formulas was common. Two examples are discussed below.

(1) Misuse of the relationship $v = \lambda \nu$. When attempting to relate the speed and de Broglie wavelength of an electron, many students used the equation $v = \lambda \nu$, which relates phase velocity, wavelength, and frequency. They tended to identify the phase velocity in this formula with the velocity of the particle. Students rarely articulated what they believed the frequency ''ν'' represented with regard to electrons or other particles. A few seemed to interpret ''frequency'' as the number of particles that reach the screen per unit time.

(i) Questions on varying the accelerating voltage, speed, or kinetic energy of the electrons. About 25% of the students in the algebra-based course and about 20% of those in the calculus-based course incorrectly used the equation $v = \lambda \nu$ to predict that increasing the speed would cause an increase in the wavelength of the electrons. Some modern physics and quantum mechanics students also made this error. Rather than recognizing that the de Broglie wavelength is inversely proportional to the momentum of the electrons (and thus inversely proportional to the speed), they incorrectly predicted that an increase (or decrease) in the speed of the electrons would result in a proportional increase (or decrease) of the wavelength.

(ii) Questions on replacing the electrons with different particles. On questions in which students were asked to compare the velocity of electrons with the velocity of particles of greater mass that produced the same pattern, some ignored the difference in the masses. These students argued correctly that since the pattern is the same, the wavelength must be the same. However, they then incorrectly used the relationship $v = \lambda \nu$ to predict that the speed of the electrons and the more massive particles would have to be equal.

Students often made a similar error on the questions in which they were asked what would happen to the pattern when electrons were replaced with particles having the same kinetic energy but different mass. Many correctly reasoned on the basis of the kinetic energy ($\frac{1}{2} m v^2$) that the heavier particles would have a smaller velocity than the original electrons. However, they then used the relationship $v = \lambda \nu$ to conclude that the more massive particles must have a smaller wavelength. Although this is the correct answer, the reasoning is incorrect.

(2) Misuse of the relationship $E = hc/\lambda$. Many students referred to the equation for the photon energy, $E = hc/\lambda$, in their responses to the problem on electron diffraction. They tended to associate the energy E with the kinetic energy of the particle. Some claimed that increasing the mass of the incident particles would have no effect on the pattern since the mass does not appear in the equation. These students applied an equation that would be valid for relating the photon energy to the wavelength of light but not for relating the kinetic energy of a nonrelativistic electron to its de Broglie wavelength. They failed to recognize that the relationship $E = hc/\lambda$ holds only for massless particles.

IV. DEVELOPMENT OF THE TUTORIAL *WAVE PROPERTIES OF MATTER*

The student difficulties illustrated above are prevalent and apparently persist beyond the first or second exposure to the material. This finding is consistent with our experience that the study of advanced material does not necessarily deepen conceptual understanding of topics taught at earlier levels.[17]

To help students overcome the difficulties that we identified in this study, we used an instructional approach that has been shown to be successful in the introductory calculus-based course.[18] The basic procedure is to make incremental, but intellectually significant, modifications to the treatment of a given topic through the development of research-based tutorials. The primary purpose of the tutorials is to engage students actively in the learning process. The emphasis is on constructing concepts, developing reasoning skills, and relating the formalism of physics to real-world phenomena, not on transmitting information and solving end-of-chapter problems. The tutorials are intended to supplement standard instruction by lecture, textbook, and laboratory.

Many of the tutorials make use of an instructional strategy that we have found effective for addressing serious conceptual and reasoning difficulties. The process can be summarized as consisting of three main steps: *elicit, confront, resolve*.[19] Since a single experience is rarely adequate to overcome a serious difficulty that has been highly resistant to traditional instruction, students must be given repeated opportunities to *apply* what they have learned in different contexts, to *reflect*, and to *generalize*.

A typical tutorial sequence consists of a pretest, worksheet, homework assignment, and a post-test. Each tutorial is preceded by a pretest that serves to *elicit* some of the conceptual and reasoning difficulties with the material that have been identified by research or teaching experience. During the subsequent 50-min tutorial sessions, students work collaboratively in groups of three or four through worksheets designed to help them *confront* and *resolve* specific difficulties. Tutorial homework assignments help students reinforce and extend what they have learned during the tutorial sessions. The material covered in the tutorials is post-tested on course examinations. Like the pretests, the post-tests emphasize qualitative reasoning and verbal explanations.

A. Context for development of the tutorial

The development of tutorials by the Physics Education Group takes place in an iterative cycle. Like all the curriculum produced by our group, the tutorials are designed, tested, and revised with the target populations. For *Tutorials in Introductory Physics*, most of this process takes place in the calculus-based course at the University of Washington. Since the tutorials are an integral part of this course, there are many opportunities for ongoing assessment. Pilot-testing at other universities and colleges provides feedback that can be used to help increase the adaptability of the materials in different instructional settings.

The situation for the tutorial *Wave Properties of Matter* has been somewhat different from that described above. Modern physics is not usually treated in introductory calculus-based physics at the University of Washington. (The classes discussed in this paper were exceptions.) Although modern physics is taught in the algebra-based course, there is no tutorial system to facilitate repeated testing of curriculum in a systematic way. In the second-year and third-year course, the development and testing of a tutorial is dependent on the interest of the individual faculty member teaching the course in a particular academic quarter.

Design and testing of the tutorial has taken place in the context of a few classes at all four of the instruction levels discussed in the paper: the introductory algebra-based course, the calculus-based course, the second-year course in modern physics, and the third-year course in quantum mechanics. Although development has taken place over a period of several years, the number of opportunities to test and revise the tutorial is relatively small. However, the preliminary results are consistent and sufficiently promising to warrant a discussion of our instructional approach.

B. Description of the tutorial

The tutorial *Wave Properties of Matter* is designed to help students deepen their understanding of a wave model for light and extend it to a wave model for particles. An important goal of the tutorial is to help students recognize the relevance of the de Broglie wavelength and to apply it correctly in accounting for changes in the location of interference fringes. It is also intended to help students overcome some specific difficulties with the de Broglie wavelength that do not seem to be adequately addressed by the standard treatment of this topic.

The tutorial is preceded by a pretest. (The questions described earlier served as pretests for the classes in this study.) The tutorial worksheet consists of two main parts. The first is in the context of double-slit interference of light; the second is in the context of double-slit interference of electrons. In the first part of the tutorial, a series of questions helps students review relevant concepts from physical optics, such as superposition and path length difference (or phase difference). In the second part, the students make an analogy between double-slit interference of light and of electrons. They are led to recognize the relationship between the momentum of the electrons and their de Broglie wavelength. The tutorial worksheet is supplemented by a homework assignment that provides an opportunity for additional practice and reflection.

1. Review of double-slit interference of light

At the beginning of the tutorial, the students are shown a double-slit interference pattern for light. (See Fig. 2.) They are led to recognize that the light through each slit reaches the entire screen and that the fringes arise from constructive and destructive interference. The students are guided through the reasoning required to derive the equations that express the angles to the interference maxima and minima in terms of the slit separation and wavelength. They are then asked to determine the form of their equations in the limit that the angles are small ($\sin\theta\approx\theta$). They use these equations (e.g., $d\theta\approx m\lambda$ for interference maxima) later to quantify changes in the pattern on the screen.

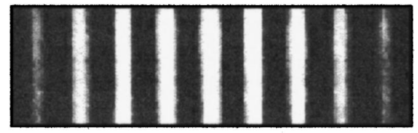


Fig. 2. Photograph of a double-slit interference pattern used in the tutorial *Wave Properties of Matter*.

The students are then shown two interference patterns, in which the maxima and minima are slightly farther apart in the second than in the first. They are told to assume that only one change was made to the experimental setup. The students are asked whether a change in slit separation could have been responsible for the change in the pattern, and if so, to determine whether the slit separation would have increased or decreased. By reasoning on the basis of path length difference, or by applying the equations that they have derived for the interference maxima (or minima), they conclude that the slit separation would have decreased. The students are then asked whether a change in the wavelength could have been responsible for the change in the pattern, and if so, to determine: (a) whether the wavelength would have increased or decreased, and (b) whether it would have changed by a factor greater than, less than, or equal to 2. (A specific numerical factor is used in anticipation of the next part of the tutorial.) By using the equations they derived for small angles, the students determine that the wavelength must have increased by the same factor as the angles, and thus by a factor of less than 2.

2. Consideration of double-slit interference of electrons

The second part of the tutorial deals with electrons that have been accelerated from rest through a known potential difference V_0 and are incident on two very narrow slits. The students are shown a photograph of the pattern on a phosphorescent screen placed far from the slits. They recognize that the presence of maxima and minima suggests that the electrons are exhibiting wave-like properties.

The students are asked to predict how, if at all, the locations of the interference maxima would change if the accelerating voltage V_0 (and thus the kinetic energy) were halved. This question is intended to *elicit* incorrect responses from students who have difficulty relating the de Broglie wavelength of an electron to its speed, momentum, or kinetic energy. A specific numerical factor is given so that we can determine whether students attribute to the wavelength the correct functional dependence on the momentum. Various errors will lead to different predictions. One common error is to claim that the maxima would stay in the same place but become dimmer. Some students recognize that the speed of the electrons would decrease but incorrectly apply the equation $v=\lambda\nu$ to predict that the wavelength of the electrons would also decrease. Other students realize that the maxima would move farther apart but predict that the angles to the maxima would increase by a factor of 2.

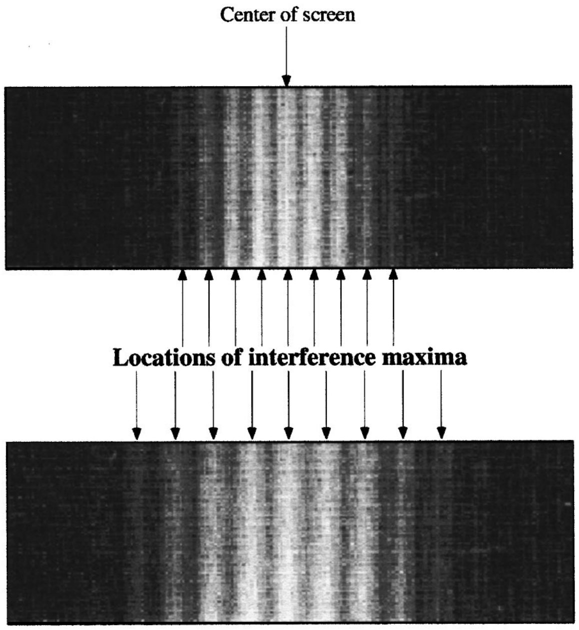


Fig. 3. Figure from a student handout used in the tutorial *Wave Properties of Matter* that shows the interference patterns made by electrons before and after the accelerating voltage is halved.

Next, the students are led to *confront* any errors that they may have made in their predictions. They are given photographs that show the interference patterns made by electrons before and after the accelerating voltage is halved. (See Fig. 3.) By comparing the photographs, they observe that halving the accelerating voltage causes the angles to the interference maxima to increase by a factor that is less than 2.

Finally, the students are given the opportunity to *resolve* any inconsistencies between their original predictions and the photographs. On the basis of the patterns shown in the photograph and the equation (in the small-angle limit) for the interference maxima, the students infer that the wavelength has increased by a factor of less than 2. They then determine that halving the accelerating voltage causes the kinetic energy to decrease by a factor of 2 and the momentum to decrease by a factor of less than 2. Thus the students confirm that the increase in wavelength and decrease in momentum are consistent with the definition of the de Broglie wavelength $\lambda = h/p$.

C. Use of the tutorial

The tutorial *Wave Properties of Matter* was used in somewhat different ways in the four courses involved in this study. In the algebra-based and calculus-based courses, the pretest was given in the first 10 min of one of the lectures. The tutorial itself was conducted during a subsequent 50-min small-group session, in which students worked collaboratively in groups of three or four. As the students progressed through the worksheets, the tutorial instructors helped guide them through the necessary reasoning by asking additional questions. The procedure was similar in the modern physics and quantum mechanics courses, but the 10-min pretest and the tutorial both took place in a lecture hall during the time period that ordinarily would have been devoted to a 50-min lecture.

D. Assessment of the tutorial

Some of the written problems described earlier have served as tutorial pretests in the classes included in this investigation. For post-tests, we have administered problems similar in nature to the pretests but sufficiently different that students cannot give correct answers on the basis of memorization. We have compared the pretest and post-test performance of students at all three levels of instruction. We have also compared differences between pretest and post-test performance at each level according to the class standing of the students (lowest to highest quartiles).

Single-slit diffraction of particles has been the context for all but one post-test. Since the tutorial deals only with double-slit interference, the students must extend what they have learned to a different situation. In previous studies, we had found that students often do not treat single-slit diffraction as an interference phenomenon.[4] Therefore, we decided that diffraction would provide a sufficiently different context for the post-tests.

As was the case with the pretests, each post-test consists of two or more questions. The first (type S) asks students to predict the effect on a given diffraction pattern of changing the slit width. One or more additional questions (type P) probe their understanding of the de Broglie wavelength. The students are asked to predict the effect of changing the speed or kinetic energy of the electrons, or of replacing them with particles of different mass. As on the pretests, the terms ''wavelength'' and ''de Broglie'' wavelength are not used.

The amount of time devoted to the de Broglie wavelength varied in the classes involved in this study. In some cases, the tutorial replaced a lecture on this topic. In other cases, use of the tutorial resulted in students spending additional time on the de Broglie wavelength (~50 min in tutorial plus time spent on the homework). The post-test results were similar in all cases and have been combined.

1. Comparison of results for type S questions

In all courses, there was an improvement on the type S questions (in which changes were made to the slit width, slit separation, or lattice spacing). [See Tables I(a) and I(b).] Many of the students seemed to have overcome the specific difficulties that we had tried to address.[20]

The percentage of students answering the type S post-test question correctly was greatest in the calculus-based classes. About 95% answered correctly, with about 90% giving correct reasoning. [See the fourth column of Table I(b).] These students had previously worked through the tutorial series on physical optics.[14] The brief review of double-slit interference in the *Wave Properties of Matter* tutorial apparently helped them apply what they had learned in the context of physical optics to the case of interference or diffraction of particles. The students in the algebra-based classes and in the modern physics and quantum mechanics classes also seemed to benefit from the review of double-slit interference at the beginning of the tutorial. The percentages of correct responses in these courses ranged between 60% and 85% with the percentages giving correct reasoning between 40% and 65%.

2. Comparison of results for type P questions

There was an improvement in student performance on each of the type P questions. Table II(b) gives results from the questions in which changes are made to the accelerating voltage, speed, or kinetic energy. Table III(b) gives results from the questions in which the electron is replaced by a particle with a different mass. The modern physics and quantum mechanics classes are combined in each table since their performance was similar. In the algebra-based course, only the question on replacing electrons with particles of different mass was given as a post-test. In all other courses, different versions of both the type P questions have been asked before and after tutorial instruction. Since the results from the various type P questions are similar, only the version in which the mass of the incident particle was changed are discussed below (Table III).

In the algebra-based class, only about 20% of the students had answered correctly on the pretest after standard instruction, with about 5% giving correct reasoning. After working through the tutorial, about 85% answered correctly, 65% with correct explanations. The students in the calculus-based course had performed similarly to the algebra-based course on the pretest. About 20% had answered correctly, 10% with correct reasoning. On the post-test, about 65% answered correctly with 55% giving correct reasoning. The percentage of students answering correctly in the modern physics and quantum mechanics classes rose from 35% on the pretest to about 65% on the post-test. When reasoning is taken into account, the percentages for these classes increased from about 25% to about 55%.

Students in all the classes seemed to have benefited from the tutorial. From Table III, it appears that the gain in the algebra-based course is greater than that in the other courses and that the students in the calculus-based courses performed as well on the post-test as those in the modern and quantum physics courses. There are several possible explanations. We have evidence that the version of the type P question given as a post-test in the algebra-based course is somewhat easier than the versions given in the other courses. That version has also been given as a pretest in a calculus-based class (not discussed in this paper since the students had not been given a post-test) and about 45% of the students gave correct answers. Another possible factor is that in the algebra-based and calculus-based courses discussed in this study the tutorials were an integral part of the course. The students had worked through several other tutorials prior to working through the tutorial *Wave Properties of Matter* and had done so in a room with small tables at which they could work easily in groups of three or four. In the modern physics and quantum mechanics classes, the tutorial *Wave Properties of Matter* was the first one used in the class. Moreover, because the pretest was given immediately before the tutorial, the students had only 40 (instead of 50) min to work through the tutorial. Also, the tutorial took place in the lecture hall, where it was difficult for the students to work together and for the instructor to engage them in small-group discussions.

3. Effect on students of different academic achievement

We have tried to assess the effectiveness of the tutorial on students of different background and ability in physics. We divided each class into three groups according to academic achievement in the course: the top 25%, the middle 50%, and the bottom 25%. The criterion was performance on homework and examination questions on material not specifically covered in the tutorial *Wave Properties of Matter*. We then examined the pretest and post-test performance of students in each group.

Students who answered the pretest incorrectly were from all academic levels. After working through the tutorial, a significant portion of the students from the top, middle, and bottom parts of all the classes answered the post-test correctly. Similar trends were observed in all the classes.

Our informal observations of students indicate that even students who respond correctly on the pretest appreciate the challenges presented in the tutorials. As they struggle with the subtleties, they arrive at a deeper conceptual understanding than they would have otherwise. We have noted a similar effect on the graduate students who serve as instructors in the tutorials.[21]

V. CONCLUSION

Helping students understand the wave–particle duality is an important goal of instruction in modern physics. As a prerequisite, students must be able to apply a basic wave model to account for wave phenomena, such as diffraction and interference. In the present investigation, however, we found that many students in first-year, second-year, and third-year physics courses had not developed this ability during their study of physical optics. Another prerequisite that they lacked was an understanding of the role of the de Broglie wavelength. At all levels of instruction, many students did not recognize its relevance to the diffraction or interference pattern or understand the dependence of the de Broglie wavelength on the momentum. Instead, they tended to treat the de Broglie wavelength as a fixed property of a particle.

The tutorial *Wave Properties of Matter* was developed to help students learn to apply the de Broglie wavelength to account for the locations of the maxima (or minima) in an interference (or diffraction) pattern and to recognize the dependence on the momentum. In the present investigation, we identified specific student difficulties with the wave model for matter and with the relationship of the de Broglie wavelength to the model. This information was crucial in guiding the design of strategies to address specific student difficulties.

In all classes that worked through the tutorial, there was an improvement in performance on questions that probed student understanding of these issues. The relatively short time involved seems to have helped students overcome the difficulties that the tutorial was designed to address. Students of different academic achievement and at all levels of instruction benefited.

The fact that similar results were obtained from the various classes that participated in this study suggests that it was not the time spent on task that determined the outcome. Nor was class size critical. Both of these inferences are supported by research on student understanding of topics other than the de Broglie wavelength. At the University of Washington, the tutorials have replaced one lecture each week. At other colleges and universities, the tutorials have replaced problem-solving sessions. In both situations, there is evidence that students who have worked through the tutorials have developed a deeper understanding of the relevant material than those who have not.[22,23] Other investigators have similarly found that the time spent on a task is not the determining factor, even when the lecturer has explicitly tried to address known student difficulties.[24]

It has been our experience that serious conceptual difficulties are seldom overcome through listening to lectures and solving standard problems. We have found that an effective instructional approach is to require students to go through the chain of reasoning necessary for the development and application of important concepts. The tutorials represent an approach for achieving this goal that has been extensively tested.

Other aspects of the diffraction and interference of matter have not been explicitly addressed in the research reported in this paper or in the tutorial that has been described. Examples include student understanding of the formation of the pattern by many individual particles (one at a time) and the interpretation of the pattern as a probability distribution.[25] Additional research is necessary to provide a guide for the design of instruction to help students deepen their understanding of these and related phenomena.

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[7]For an article that illustrates the development and assessment of a tutorial in geometrical optics, see Ref. 2. For an example in physical optics, see K. Wosilait, P. R. L. Heron, P. S. Shaffer, and L. C. McDermott, ''Addressing student difficulties in applying a wave model to the interference and diffraction of light,'' Phys. Educ. Res., Am. J. Phys. Suppl. **67**, S5–S15 (July 1999).

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[13]See, for example, Ref. 2.

[14]This series includes tutorials on two-source interference, double-slit interference, multiple-slit interference, and single-slit diffraction. See Ref. 6.

[15]Some of the students in the advanced courses had used the tutorials on interference and diffraction of light in their introductory classes. However, only a few had worked through the same revised version that the students in the calculus-based course had used. Although those students seemed to do better than the students who had not worked through that version or who had not used the tutorials at all, the number of students was too small to draw definitive conclusions about significant long-term retention.

[16]Just before publication of this article, type P questions were administered in two calculus-based courses before lecture instruction on the de Broglie wavelength. (The students had recently completed the tutorial sequence on physical optics.) The percentage correct and the percentage with correct reasoning were within 5% of the results for calculus-based students after lecture instruction. (See Tables II(a) and III(a).

[17]In addition to Refs. 2, 4, 5, and 7, see L. C. McDermott, P. S. Shaffer, and M. D. Somers, ''Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine,'' Am. J. Phys. **62**, 46–55 (1994); T. O'Brien Pride, S. Vokos, and L. C. McDermott, ''The challenge of matching learning assessments to teaching goals: An example from the work-energy and impulse-momentum theorems,'' *ibid.* **66**, 147–157 (1998).

[18]For a description of the tutorial system at the University of Washington, see the articles in Ref. 17.

[19]For a discussion of this instructional strategy, see L. C. McDermott, Millikan Award Lecture: ''What we teach and what is learned—Closing the gap,'' Am. J. Phys. **59**, 301–315 (1991).

[20]We consider a tutorial successful when the post-test performance of introductory students matches or surpasses that of tutorial instructors (graduate students and advanced undergraduates) on the corresponding pretests. Type P questions were given to more than 25 TA's enrolled in a graduate teaching seminar. About 80% correctly predicted the effect of changing the speed of the electrons (70% with correct reasoning). When the electrons were replaced with other particles of the same kinetic energy, about 50% gave the correct response (40% with correct reasoning). The post-test performance of students at all levels matched or surpassed that of the TA's on the pretest. (See Tables II(b) and III(b).)

[21]See, for example, the last article in Ref. 17 and P. S. Shaffer, ''Research as a guide for improving instruction in introductory physics,'' Ph.D. dissertation, Department of Physics, University of Washington, 1993 (unpublished).

[22]See for example, Ref. 4 and the first paper in Ref. 17.

[23]*Tutorials in Introductory Physics* is being pilot-tested in situations in which the tutorials replace standard problem-solving sections. Typically, we have found that the tutorial students not only perform much better on qualitative questions but as well, or better, on related quantitative questions. The result is consistent with those obtained by G. E. Gladding, University of Illinois, D. Elmore, Purdue University, and E. Mazur, Harvard University (private communications).

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[25]See Ref. 10.