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## Student difficulties in learning quantum mechanics

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For university students studying physics, quantum mechanics is considered an extremely difficult subject, but one which must be taught increasingly early in their careers. We report a preliminary project which used a phenomenographic approach to explore the ways in which a small number of fundamental ideas are conceptualized by students who have been judged to have successfully mastered the material. The results suggest that the mental models used by these students are technically advanced but structurally unsophisticated. Whether a change in current teaching practices might lead to better conceptualization is a question that needs further exploration.

### Introduction

Quantum mechanics is an area of physics which is, above all others, of immense importance in modern technology – lasers, transistors and semiconductors are but a few applications. Students who are studying physics for professional reasons need to know this subject well. However, the concepts involved in it are complex and counter-intuitive. **They need a lot of time and reflection to be absorbed properly.** Therefore it would be desirable for students to **meet these ideas early** in their career, even in high school if possible. Unfortunately, quantum mechanics is also a subject which most students traditionally find very abstract and difficult, and its **teaching has not changed** much since it was invented early this century. It is an area which has **not, until recently, attracted much pedagogical research<sup>1</sup>** and it is timely that university teachers should be investigating ways in which it might be taught more effectively.

There are (at least) two difficulties facing such an investigation. First, the subject is shrouded in a highly mathematical formalism, and, though some textbook authors have sought to simplify the demands this makes on students, there is **not yet consensus about how it might be taught less abstractly.** Second, the subject is in a state of flux – **questions of how the formalism should be interpreted are still discussed in the technical literature<sup>2</sup>.** The project reported here proposes a line of enquiry that seeks to **answer two fundamental questions.**

- (1) Is it possible **to identify the most important concepts** that students need to understand in order to learn quantum mechanics ‘successfully’?
- (2) What is it about the **way students conceptualize** the ideas of quantum mechanics which makes them particularly ‘difficult’?

In this paper the results of a preliminary study are reported. A **small number of concepts were selected**, which experience suggested were basic to an understanding of the subject. A class **of students who have already studied this subject** successfully, as success is usually measured, were surveyed in order to bring to light differences in how they conceptualized the material. The ultimate aim is that analysis of these responses should lay the groundwork for a further, more focused study.

## Theoretical background

### *The learning context*

The educational context in which today's students learn about quantum mechanics is very different from the social environment in which scientists first postulated and then developed this way of describing the behaviour of matter on a microscopic scale. It took nearly 30 years for the original theories to be developed. Physicists struggled to understand and refine new ideas and concepts that were initially highly contentious and based on abstract philosophical and mathematical principles. In contrast, most students of physics entering university courses today do so with an experience of school physics in which approved 'knowledge' has been learned as scientifically 'correct', successfully encoded from teachers' notes and textbooks and reproduced for examinations at the matriculation level. School physics also has a strong Newtonian flavour, in the sense that, although most students' initial experience of physical models of reality may be somewhat counter-intuitive, these models do explain the behaviour of objects within the range of normal sensory experience.

Vygotskian approaches to learning (Vygotsky 1978, Newman *et al.* 1985, Wertsch 1985) emphasize the **social ontogeny of knowledge**, the non-absolute nature of knowledge and the relationships between knowledge and experience. Vygotsky describes Activity as a process undertaken in a social context, in which groups or individuals grapple with new information or new demands to make meaning, to resolve problems and to adapt to new conditions.

Rogoff (Rogoff 1994) describes educational settings as communities of learners in which different forms of social organization and intellectual activity result in different forms of knowledge, skills and capabilities. In such a systemic view, many kinds of activity are associated with learning. Brousseau (Brousseau 1992) suggests that **a conceptual shift occurs when knowledge is 'taught' in courses in educational institutions away from the context of activity where the knowledge first evolved.** He makes the point that **attempts to obtain knowledge independently of situations where it works (decontextualization)** usually result in loss of meaning and performance of the time of teaching.

Thus, teaching activities have an impact on the ways teachers conceive of their subject domain. In traditional settings, in which a transmission model of learning is assumed, students' 'success' is defined in terms of accurately encoding information and reproducing it on demand for assessment tasks. In a context of this kind, success for teachers lies in facilitating the above processes. In professional or research settings, where an ability to apply concepts and principles is at a premium, experience in troubleshooting and problem solving results in knowledge

that can be applied in the interpretation, definition and resolution of other related problems.

The **social context for learning physics** in undergraduate courses at university is, in general, one in which course designers tend to compartmentalize the learning context, **divorcing that part of the subject which the scientific community is agreed upon from the processes of evolving and testing models that is undertaken by professional scientists.** Textbooks and lectures, particularly in the earlier years, still provide 'correct' information. Students are required to demonstrate that they have encoded the information accurately, can reproduce essential facts and ideas in examinations and can apply physical models to solve problems. **Such a context favours cognitive processes associated with encoding and reproducing information. It is not conducive to reflection and review, nor to the construction of personal meaning** that is necessary to develop new concepts or a new schematic lens through which to interpret the physical world. **Assessment tasks usually assume an absolutely correct answer even though the scientific mental models that are the focus of study are constantly under review.**

Successful learning is defined in terms of correctly completed assessment tasks that demonstrate a 'correct' interpretation of course 'content'. These traditionally involve paper-and-pencil activities that are completed under pressure of time. Teachers of physics at university level, even those actively researching in the field, come to view well-established topics in terms of their experience as providers of information and decision making about the best textbook to use, how best to convey key ideas to groups of students and how to check that they are 'understood'. In such a context, 'difficulty' is noticed when students are unable to perform assessment tasks. When many students are unable to complete tasks according to teacher expectations of 'correctness', the topic or physical concepts are considered to be difficult.

Quantum mechanics is a good example of a field where students experience this kind of difficulty. Learning about quantum mechanics involves a fundamental reconceptualization or shift in intellectual activity in many different areas. In thinking about quantum mechanics students must **move beyond models based on sensory experience towards models that encapsulate theoretical sets of abstract properties.** It may be expected that if the context of learning does not promote the kinds of activity that foster conceptual development and personal involvement in meaning making and remaking, then students will fail to develop adequate mental models as a basis for reasoning, researching and problem solving in this field.

### *The physics context*

Quantum mechanics was developed in the period (roughly) 1900–30. It grew out of a series of subtle experimental observations which are, even today, outside most people's normal experience. It was put together by a group of scientists of formidable mathematical expertise, and though several seemingly different 'representations' of the subject have appeared subsequently, this **mathematical bias** still persists. Historians of science judge that this development marked a radical change in scientific thought – from 'Newtonian' to 'modern' physics – and they often focus on **two particular items** as symbolic barriers that had to be surmounted: the **wave-particle paradox and the Heisenberg uncertainty principle.**

The wave-particle **paradox** can be traced back to the historical argument about whether light was made up of small *particles* which moved in straight lines, which many scientists before 1800 thought; or whether it propagated like *waves* in some unseen ‘ether’, which experiments in the early 19th century conclusively supported. At the beginning of the 20th century new experiments suggested that, in some circumstances, *both* ways of thinking about light could be necessary. In the 1920s a similar conclusion was drawn about the atomic constituents which made up ordinary matter. Initially they were observed to act like particles – they registered as points on a photographic film, for example – but again, newer experiments demonstrated they also showed wave behaviour. **From the point of view of those enmeshed in the historical controversy** in which waves and particles were somehow in opposite camps, these observations were considered **paradoxical**.

For the project that is reported here, the wave-particle paradox seems, *prima facie*, to be a concept that students would need to be clear about. There is, of course, **no real paradox**. Either particle or wave is simply a *model* defining a set of properties which we correlate with the set of experimental results we wish to explain.<sup>3</sup> Clearly, quantum mechanical behaviour demands a new model which combines some of the properties of each. Nevertheless, the writers of textbooks on quantum mechanics usually discuss the paradox before developing this new model (encapsulated, mathematically, in the Schrödinger wave equation). There seems to be implicit consensus that students need to be aware that they are working only with models of reality, and that these models have limitations. Indeed, as the subject develops further, they will be expected to work with different mathematical representations of the basic model. It seems, therefore, a valid question for this project to ask:

**How do students of quantum mechanics conceptualize the mental models involved in the wave-particle paradox?**

The second basic concept mentioned above is the uncertainty principle, which **can best be understood** in historical terms as an attempt to solve the wave-particle paradox by postulating that any microscopic object sometimes behaves like a particle and sometimes like a wave, but most of the time it is ‘in between’. It was Heisenberg who gave formal meaning to this idea by focusing on the physical act of measurement of, say, a wave property like frequency. **When the object is acting like a wave, the measurement of its frequency is predictable and unambiguous; but for the rest of the time, the result of the measurement cannot be predicted with certainty.** This implies that, even though any one measurement must yield a definite value for the frequency, if the measurement were repeated it would not necessarily give the same result. All measurements have an associated uncertainty.

The *reason* for this uncertainty is a philosophical question about which there is still argument. The most widely accepted view is that it does not arise because of inescapable inaccuracies in the measuring apparatus, which is the context in which most physics students first meet the word ‘uncertainty’. Nor does it arise because we may not know enough detail to say precisely what will happen – in the way, for example, that the weather is considered to be ‘unpredictable’. Instead it is believed that, at a microscopic level, it is intrinsically impossible to predict what will happen in any situation. There is, in nature itself, a fundamental indeterminacy. Yet the equations of quantum mechanics are deterministic mathematical statements. Under well-defined circumstances, these equations will make well-defined,

numerical predictions. It follows, then, that such predictions can only be interpreted *probabilistically*.

For students learning quantum mechanics, all this poses great difficulties. The mental models they have worked with before, wave or particle, were *pictorial* models. They learned to create images, or draw pictures, to help conceptualize various ideas. The new quantum mechanical model requires a further level of abstraction. The most commonly used image, the wave packet, cannot in any sense be considered to 'look like' the object. It is merely a device for predicting the object's physical behaviour. Bringing students' thinking to this level of sophistication is one of the most difficult aspects of teaching the subject. They must understand the fundamental concepts which make such an approach necessary. The distinction between uncertainty as a measurable property of any object and indeterminacy as an interpretive concept is one which was very clear to those who invented quantum mechanics. Hence the current project also focuses on the question:

How clearly do students organize their thinking about abstract concepts like uncertainty and indeterminacy?

### Survey procedures

The research reported here was carried out to explore the ways in which these ideas are conceptualized by students who have been judged to have 'successfully' mastered them. The students were asked to provide written responses to a set of questions. The responses were then analysed in a number of different ways (see later for details).

#### *Student sample*

A class of students in their third year of undergraduate work was selected for study. These students had already completed a short (12 lectures) module called Modern Physics in their first year, and a longer (18 lectures + nine computer labs) module on Quantum Mechanics in their second year. In the Australian university system it is common that only 20% of first-year students in mainstream physics courses (i.e. not in service courses) normally enrol in that subject in the second year, and 60% of those completing the second year enrol in the third year. By any standards, therefore, these were 'successful' students.

The class chosen started the year with 33 students. They then divided themselves into two roughly equal classes, one of which (17 students) studied quantum mechanics in a formal, mathematically oriented approach. The other class (16) studied the subject in an applied, experimentally oriented approach. The whole class of 33 was surveyed at the start of semester before they did the third-year course. At the end of semester, only the formally oriented class of 17 was surveyed.

For the purpose of the survey being done, the responses of these two groups of students were simply combined and analysed as though they were one class of 40 independent students. This was justified *a posteriori* on the grounds that there was no obvious difference between the three different groups – formal/before, formal/after and applied/before. Indeed this is consistent with the fact that the questions being asked most directly concerned material which all these students had started

studying two years previously. The group of 40 could then be thought of simply as a mixed group of students at various stages and in various streams of a third year course.

### *Nature of the survey*

Each survey was conducted by asking the students to answer a quiz, which consisted of a single (two-page) sheet of short-response questions. The first quiz was given at the start of the third-year course, and contained nine questions. The second was given at the end of the course and contained seven. Each quiz was distributed towards the start of a standard lecture, and students had 25 minutes to complete their answers. They were told that they could talk with one another, though in fact very few did. It was stressed that there was no formal assessment taking place. The quiz sheets were collected 10 minutes before the end, and a brief discussion of the 'correct' answers ensued.

Each question on the sheets addressed a different topic of a very general nature, which had been discussed in previous courses and in the current year. They covered, *inter alia*, the wave-particle paradox, the nature of eigenfunctions, the meaning of potential wells and the idea of operators. Two questions, however, directed towards the idea of a mental model of wave or particle, appeared on both quizzes in slightly different guises; and one specific question devoted to the concept of probability was asked on the second quiz only. The responses to these questions were the ones that were singled out for thorough analysis. The exact wordings were as follows:

From the first quiz:

- (1) We say electrons, protons and such are particles. What would you say are the simplest 'particle-like' properties that one of these things could show?
- (2) What are the defining properties of a wave? Could something be considered to be a wave if it doesn't have an identifiable wavelength?
- ...

From the second quiz:

- (1) Your friend has read about the early experiments with cathode rays and radioactive emissions and so on; and that these are now believed to be particles. What are the properties that these things show that led to their being labelled as 'particles'?
- (2) Your friend has also read that 'quantum mechanical particles can also exhibit wave properties'. What would you say are the defining properties of a wave?
- ...
- (5) Your friend has also come across the statement 'quantum mechanics is a probabilistic theory'; but finds great trouble distinguishing between the two concepts of 'indeterminacy' and 'uncertainty'. Is there a difference or do they mean the same thing?

It is worth pointing out that, even though what is being reported here is a preliminary project, the form of these questions had already been through earlier versions (see Appendix A for details).

### Analysis of responses

It was immediately obvious that two quite different approaches could be taken towards analysis of the responses. That these two approaches should closely parallel two different views of the goals of the learning process is probably not unexpected, given that the authors of this project come from different academic backgrounds.

The first method of analysis is interested in examining how the students think about the concepts involved in these questions. It seeks to categorize the responses into qualitatively different groupings, based solely on the data contained in what the students say or write. This approach draws on the work of Marton (Marton 1989), and we will describe it as a phenomenographic analysis. We believe this approach reflects a view that the aim of teaching and learning is to allow students to construct their own meanings and ideas from experiences and activities provided. Such an approach may, for convenience, be thought of as 'constructivist'.

The second approach is conditioned by the original motivation of the project. Even in the absence of complete agreement on how quantum mechanics should be interpreted, one of the basic aims of teaching it at an undergraduate level is that students should know those parts of the subject that are agreed upon. From this point of view, it would be a nonsense to ignore the *appropriateness* of what is in the students' responses. Therefore a content/correctness analysis was also planned. This approach reflects another view of the teaching process, widely held in physics (and other) departments, that students should learn to master the material early, and that its meaning will somehow 'fall into place' later. Especially in its assessment practices, which demand students be able to reproduce what is considered correct, such a view may conveniently be thought of as 'transmissionist'.

It is well known that there are many points on which these two views of the teaching process are in conflict. Therefore special interest was given to the question of how the two analyses related to one another. A correlation between the two sets of results was planned.

Initially a meeting of the three authors, two of whom were physicists and one an educationalist, was held to discuss preliminary ideas about how analysis should proceed. The following scheme was developed during a number of meetings.

#### *Stage 1: Phenomenographic analysis*

The responses were examined independently by each of the authors in order to identify a provisional set of categories. Responses to question 1 ('What is a particle?') and question 2 ('What is a wave?') revealed a very clear set of categories of description. On the other hand, responses to question 5 ('What are the meanings of indeterminacy and uncertainty?') covered a diversity of ideas and revealed only one fragmented category.

After some discussion, a final set of categories and an associated set of shared meanings were developed for questions 1 and 2. The two physicists then independently re-categorized all the responses in accordance with the (agreed) shared meanings. These analyses were compared and any differences resolved by discussion.

This phenomenographic analysis of the students' written responses resulted in the development of two sets of categories of description, one for each of the first

two questions. They provide a preliminary mapping of the students' perceptions of particles and waves in response to the questions posed within the context of quantum mechanics.

### *Stage 2: Content/correctness analysis*

In light of the importance of terminology in this subject, and the primacy of standard textbooks in determining what is important and what is not, an analysis of the responses' contents was also undertaken. A record was made of the ideas and terms presented by the students, which were then categorized according to whether they were judged, by the physicists, to be 'appropriate' or 'inappropriate'.

A *marking scheme* was then developed for the questions asked, based on the experience of the physicists, which would enable each response to be assigned a score (see Appendix B).

## **Results of analysis**

### *Phenomenographic analysis*

*Question 1:* The analysis described above, when applied to the question 'What is a particle', revealed three clear categories of description of student responses, which might loosely be paraphrased as follows:

- *Category 1:* A particle is made of stuff.
- *Category 2:* A particle is made of stuff and it travels along a well-defined path.
- *Category 3:* A particle is made of stuff and it travels along a well-defined path and it also responds to external forces.

This categorization is a hierarchical one, in the sense that category 2 responses logically, and in most cases explicitly, include category 1. Likewise category 3 includes category 2. Because each category clearly encompasses more than the previous one, we believe that categorization can be interpreted as a hierarchy of student awareness. Details of the categories, as well as representative responses in each, are presented in table 1.

Structurally the responses in all three categories were fragmented, in the sense that they consisted largely of isolated facts. In the terminology of Biggs and Collis (1982), responses in all categories were multistructural (with a number of *nodes* between *cue* and *closure*) but exhibited few relational components. The overall sophistication of the answers varied even within categories, but this sophistication was reflected mainly in the number of nodes rather than meaningful relationships between them.

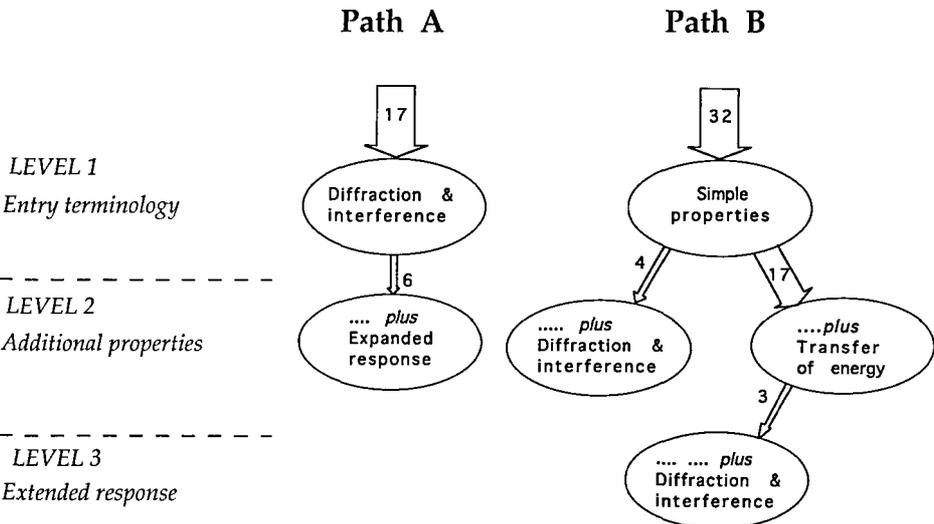
One point emerged which we believe might be important. There seemed to be a significant shift in conceptual development between the responses in categories 1 and 2, and those in category 3. Students whose responses were in the former group seemed to conceive of a particle as an isolated entity; the latter thought about it in terms of its interaction with the outside world. A similar dichotomy would emerge in question 2.

**Table 1. Categories of description of responses to question 1, as determined by the phenomenographic analysis.**

<i>Question 1—What is a particle?</i>		
<i>Category</i>	<i>Examples of student responses</i>	<i>%</i>
(1) <i>It is made of stuff</i> Single property (mass)	<ul style="list-style-type: none"> <li>• Fundamental charge and mass</li> </ul>	4
(2) <i>It travels in a well-defined path</i> Multiple properties Mass (assumed) + momentum and/or localization	<ul style="list-style-type: none"> <li>• These particles were labelled as ‘particles’ as they displayed properties of mass, that is cathode rays can impart momentum (although momentum is not a particle property) and do not travel at light speeds</li> <li>• The properties that show these ‘things’ are particles, they have mass hence momentum and so they can have a position in space</li> </ul>	63
(3) <i>It travels and responds to forces</i> Mass (assumed) + momentum (assumed) + localization (assumed) + responds to forces	<ul style="list-style-type: none"> <li>• When electrons are shot at atoms in a beam, they may deflect if a collision occurs, just like snooker balls</li> <li>• They have momentum (they can be used to bombard a wheel and make it spin) and mass – they obey Newton’s laws when subjected to a force. They possess properties of ‘inertia’</li> </ul>	33

*Question 2:* The question ‘What is a wave’ is different from the previous one in that there exists a standard answer in all the textbooks,<sup>4</sup> of which most of the students, having studied this subject for two and a half years, could not have been unaware. At the start of the analysis, it was immediately apparent that this produced two different *paths* along which students approached their responses. This is illustrated diagrammatically in figure 1.

One path took as its entry point the ‘standard answer’ (path A): the other sought to give a general answer to the question asked (path B). By and large,



**Figure 1. A diagrammatic representation of the two different paths by which students approached their responses to question 2.**

those taking path A simply presented the ‘answer’ with little supporting reasoning. Of the students who chose path B, 11 closed at the entry level, while 17 went on to develop their response further in a manner not unlike the apparent shift in conceptual development noted between categories 2 and 3 in question 1 – i.e. they saw the wave in relation to things outside itself, rather than as an isolated entity. At either level, only a very small number in this path closed on the standard answer. A brief summary of the categories of description found for these two paths is given in table 2.

**Table 2. Levels of categories of description of responses to question 2, as determined by the phenomenographic analysis.**

<i>Question 2 – What is a wave?</i>		
<i>Response path A</i> <i>Diffraction and interference</i>	<i>Response path B</i> <i>Simple properties</i>	
<ul style="list-style-type: none"> <li>● Will diffract, bending round edges. Will interfere with another wave constructively or destructively. Needs to have a wavelength to be considered a wave</li> <li>● Wave properties include: interference, diffraction and refraction etc. Interference properties are probably the most significant (illustrating constructive/destructive properties – particularly destructive)</li> </ul>	<ul style="list-style-type: none"> <li>● A wave would have frequency and wavelength, and also velocity. Something would not be a wave if it did not have wavelength</li> <li>● Characteristic wavelength, wave equations that accurately describe the behaviour of the system – probability clouds</li> </ul>	
<i>... plus</i> <i>extended response</i>	<i>... plus</i> <i>diffraction and interference</i>	<i>... plus</i> <i>transfer of energy</i>
<ul style="list-style-type: none"> <li>● A wave undergoes diffraction and has no definite position, but has definite momentum. Yes, well it has to have a wavelength, but not a definite wavelength</li> <li>● The ability to exhibit interference phenomena. Able to be refracted/diffracted. A wave is a continuously propagating disturbance</li> </ul>	<ul style="list-style-type: none"> <li>● A wave is defined as a periodic symmetric mathematical waveform, with interference, diffraction properties etc.</li> <li>● Defining properties of a wave – completely non-localized, has <i>one</i> value of wavelength, no mass, interference occurs after passing through slits</li> </ul>	<ul style="list-style-type: none"> <li>● Wave – transfer of energy from one point to another, defined by its wavelength and frequency</li> <li>● Frequency, carries energy through a medium</li> </ul>
		<i>... plus</i> <i>diffraction and interference</i>
		<ul style="list-style-type: none"> <li>● ‘Continuous’ energy transfer, no certain position, interference and diffraction</li> </ul>

Again the categories within each path are structurally inclusive – i.e. the response given at each level contains (effectively) the responses of the previous level together with further information. And, as before, the responses in each were fragmented, with few relational components. Interestingly, the degree of multi-structuralism (the number of nodes) did not show any increase down the categories.

Two interesting points seem to emerge.

- (1) Only half of the whole group of students closed on the standard answer (by whatever path). Half did not, even though we believe they knew that is what the textbooks would have said.<sup>5</sup>
- (2) We believe that a similar conceptual shift to that noted in question 1 was in evidence here, among the 17 responses that went down ‘path B/Transfer of Energy’ route. They clearly see a wave in relation to a larger whole, while the rest saw it as a localized concept (even when two or more waves were interfering).

*Question 5:* It was agreed by all three authors that the responses to this question were entirely fragmented, and this resulted in just one large category of description. There was little to be gained from continuing with a phenomenographical analysis of this question.

#### *Content/correctness analysis*

Thus far, all analysis has been carried out on the way students responded to the questions asked, with no notice being taken of the appropriateness of what they said in terms of how a physicist might have answered. Therefore, for reasons outlined earlier, the following content analysis was undertaken.

First, ‘ideal’ answers to the three questions were constructed by the physicists. These appear in Appendix B. Then the responses were read again, and this time each different property or attribute which the students had mentioned was examined and classified as ‘appropriate’ or ‘inappropriate’ depending on whether it appeared in the ideal answer or not. An attribute was considered inappropriate if it was incorrect or simply irrelevant.

The results of this analysis are presented in tables 3–5. For questions 1 and 2, only the ‘inappropriate’ attributes are tabulated, together with the number of students who offered each. For question 5, because it did not prove possible earlier

**Table 3. Categories of content of responses to question 1.**

<i>Attribute</i>	<i>Number</i>
Momentum	19
Shape/size	5
Discrete energy	4
Photoelectric effect	3
Reflection	2
Something quite irrelevant	1
<i>Not</i> a particle	1

**Table 4. Categories of content of responses to question 2.**

<i>Attribute</i>	<i>Number</i>
Fourier decomposition	9
Mathematical model	5
Something quite irrelevant	4
Velocity	2
Uncertainty	2
Wave equation	2
Spectrum	1

**Table 5. Categories of content of responses to question 5.**

<i>Attribute</i>	<i>Number</i>
<i>Indeterminacy:</i>	
Unpredictability	11
Measurement error	5
Hidden variables	1
Cannot predict probability	1
Probability	1
<i>Uncertainty:</i>	
Measurement error	7
Probability	4
Heisenberg	3
Measurement dist	1
Unpredictability	1

to find more than one category of description, *all* the attributes mentioned are included.

It is interesting to note which ‘inappropriate’ attribute was most commonly mentioned in response to each question. In answer to the question ‘What is a particle?’, 19 students (out of a total of 49) offered the concept of *momentum*. In answer to ‘What is a wave’ nine made some mention of the Fourier Theorem<sup>6</sup>. In question 5, 11 students (out of 17) associated the term ‘indeterminacy’ with intrinsic inability to predict the outcome of a quantum measurement, and seven associated the term ‘uncertainty’ with error of measurement.

While most physicists would consider ‘unpredictability’ to be close enough in meaning to ‘indeterminacy’ for that response to be deemed ‘appropriate’, the other three are definitely not, for the following reasons.

- *Momentum*: In quantum mechanics textbooks, position and momentum are universally juxtaposed, in the sense that position is a particle-like attribute and momentum (because of its relationship with wavelength<sup>7</sup>) is wave-like. Even in classical mechanics, momentum is something which both particles and waves possess and therefore cannot be used to distinguish one from the other. Why, then, do so many ‘successful’ students proffer it as a specifically particle-like property?

- *Fourier decomposition*: The notion that a wave packet may be built up from simple waves, or, conversely, that a complex wave may be broken down into simpler components, is enormously useful in quantum mechanics. It is also something that half the students participating in this exercise had studied in recent mathematics courses. Yet there is no way it can be considered as a *defining* property of a wave. What seems to be happening here is that students are allowing their level of mathematical sophistication to distort their responses to a much simpler question, to such an extent that they end up saying, in effect: ‘a wave is that which is made up of waves’.
- *Measurement error*: ‘Uncertainty’ is a word whose meaning is particularly context sensitive. In experimental (non-quantum) physics, it means just what these students said – that any measurement will be accompanied by some error caused by limitations of the measuring process. However, it has a different meaning in quantum mechanics – the degree to which measurements on a microscopic scale are intrinsically non-reproducible. This seems to be a clear-cut example of the difficulty students have in dissociating terminology from context.

The reasons why students chose to answer the questions as they did can, of course, only be conjectured – short of repeating the experiment and interviewing them afterwards. It is, however, obvious that in a university context marks play an important motivational role and successful students develop strategies for answering questions in such a way as to attract marks. It seemed reasonable therefore to undertake the following analysis.

Using the ‘ideal’ answers already constructed in Appendix B, a marking scheme was developed. This was done in a manner common for marking examinations in physics – i.e. key points were identified which the instructor hoped would occur and each assigned a sub-mark related to its importance. Any response which mentioned a key point (without saying it wrongly) gained the corresponding sub-mark. Each question had a possible maximum of six marks. Occasionally, but not often, submarks were deducted for totally incorrect statements. The ‘ideal’ answers and the marking schemes appear in Appendix B.

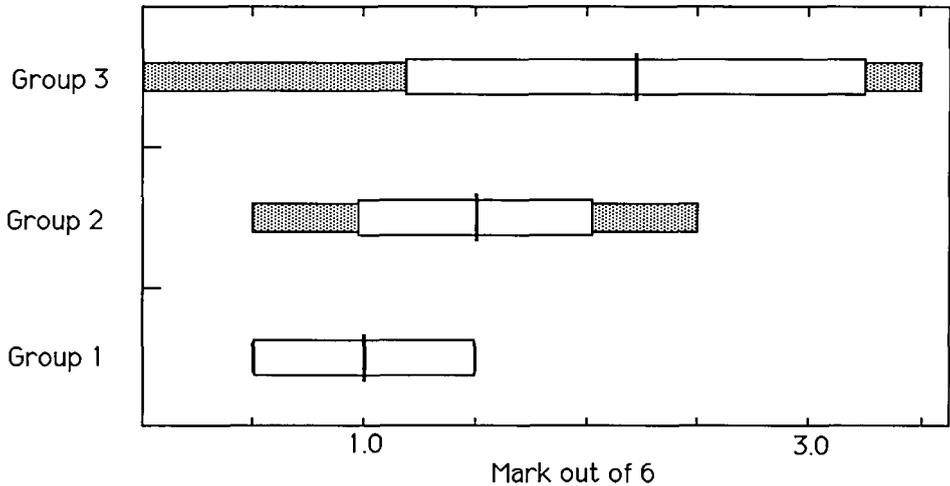
The marks awarded are summarized in table 6.

#### *Correlation between the two analyses*

Once the above analysis had been carried out, the question was asked: Is there any direct relationship between the ‘correctness’ of the response and the complexity of the model the students used to conceptualize what they were writing about? In

**Table 6. Marks awarded to each of questions 1, 2 and 5 in accordance with the marking scheme in Appendix A.**

<i>Factor</i>	<i>Question 1</i>	<i>Question 2</i>	<i>Question 5</i>
Highest score (/6)	3.5	2.5	3.5
Lowest score	-0.5	0.0	0.0
Mean	1.6	1.3	1.7



**Figure 2.** The marks for question 1 awarded to responses in each of the categories of description, categories 1, 2 and 3, in table 1. The shaded rectangles represent the range of marks awarded, the unfilled rectangles the standard deviation on either side of the mean, and the short vertical line the mean mark.

other words: is there any relation between the mark awarded and the categories of description, for either question 1 or 2?

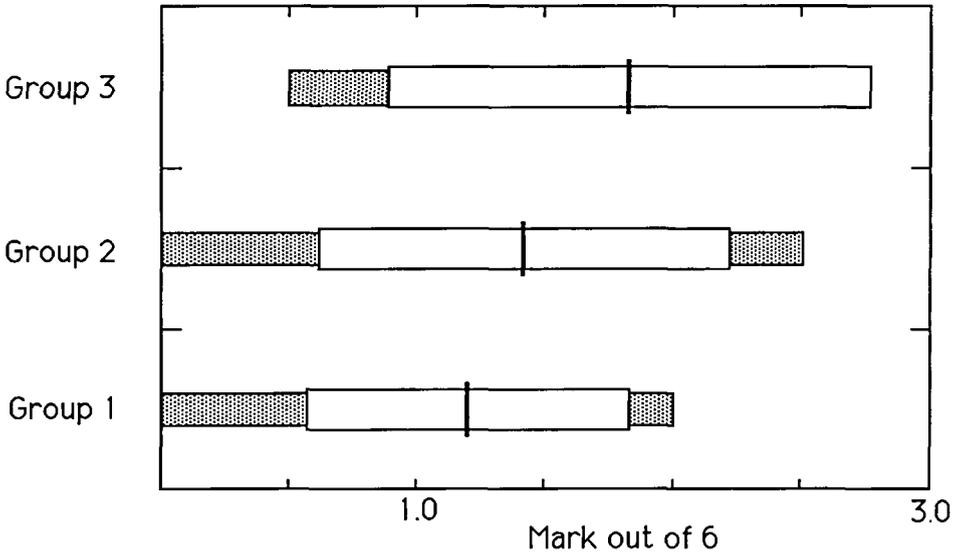
This was examined by plotting the marks awarded as follows. One graph shows the marks awarded to students for question 1 in each of the categories of description 1...3 from table 1. A second graph shows the mark awarded to students for question 2 in each of the levels of category of description 1...3 from table 2. In each graph the following items are shown for each group of students:

- the range of marks awarded (signified by the shaded rectangle);
- the standard deviation to each side of the mean (signified by the unfilled rectangle); and
- the mean mark (signified by the short vertical line).

The resulting graphs appear in figures 2 and 3.

The following comments may be made:

- Both the maximum mark and the mean increase with increasing category label. This is consistent with the fact, stated earlier, that the categories are developmentally hierarchical and inclusive, and the marking scheme awards marks simply for the presentation of specific points. The higher up the hierarchy the response, the more likely it is to have mentioned enough points to be awarded a high mark. This point will be discussed later.
- Within each category there were some responses which scored very low marks, despite our belief that those in higher categories are conceptualizing the ideas with more complex mental models. While our sample was too small to support very firm conclusions, we believe this result suggests an interpretation which is consistent with the discussion of Biggs and Collis (1982). Their findings were that the responses of schoolchildren, in answer to



**Figure 3.** The marks for question 2 awarded to responses in each of the categories of description, categories 1, 2 and 3, in table 1. The shaded rectangles represent the range of marks awarded, the unfilled rectangles the standard deviation on either side of the mean, and the short vertical line the mean mark.

surveys like these, could be expected to progress up a series of hierarchical categories *in time* as their thinking about the subject becomes more sophisticated. However, when they first make the transition from one level to the next, they tend to lose track of arguments and become confused, as though their newly acquired mental models are not yet quite under control. This phenomenon is referred to by those authors as transitional responses.

The same transitional features were observed in responses to question 2.

### Conclusions

This paper began by posing two questions of a general nature:

- (1) Is it possible to identify the most important concepts that students need to understand in order to learn quantum mechanics 'successfully'?
- (2) What is it about the way students conceptualize the ideas of quantum mechanics which makes them particularly 'difficult'?

And two further questions with a more particular focus:

- (3) How do students of quantum mechanics conceptualize the mental models involved in the wave-particle paradox?
- (4) How clearly do students organize their thinking about abstract concepts like uncertainty and indeterminacy?

Use of the survey quiz revealed categories of description which could be interpreted in terms of the mental models these questions are targeting. What is striking

is that there is little evidence that these models are any more than collections of isolated facts. They do not show much of the structure that comes from these facts having been fitted into an internally consistent framework. We have already mentioned similarities with the conceptual development of schoolchildren who still have some way to go. This is supported by the observation that many students used terminology in their answers which was not appropriate to the questions asked, and which, in some cases, side-tracked their responses completely. We believe these are all indications of the fragmented nature of the underlying mental models they are using.

The results suggest that the learning that has been observed might be described as 'surface', using the terminology commonly employed (see for example Prosser and Millar 1989). And, indeed, we have found little evidence in this research of a 'deep' approach to learning. This is of great concern because these students were about to graduate from the top physics class in a university of world-class standards. These are 'good' students on all criteria used by ordinary university physics departments to define 'goodness'. Clearly, there is need for research into the question of whether our assessment practices do not sufficiently reward deep learning.

Two points are worth making.

- Mention has been made of the fragmented nature of the responses from all these students. It is an interesting exercise to examine critically the 'model answers' in Appendix B. They took a long time to construct. Had they been written in the 5 or 10 minutes these students were given, it is doubtful that they would display much evidence of a 'consistent framework in which the facts were embedded'. And, in truth, any written response, provided it contained all the relevant 'facts', irrespective of ordering, would be awarded full marks.

The 'ideal answers' and marking scheme in Appendix B were drawn up by the two authors who were physicists. Any other physicist undertaking the same task might come up with slightly different points, and slightly different mark weightings. However, we firmly believe, based on a lot of experience in teaching university physics, that the result would not look too different. In particular the last remark would still hold, namely that it is the 'facts' which would be awarded marks, rather than any structure in which they were embedded.

It is entirely possible, therefore, that the teaching and assessment strategies commonly adopted by physics departments are encouraging exactly the kind of fragmented conceptual development being observed.

- From the similarities observed to the conceptual development of schoolchildren, it is tempting to believe that these students might be expected to 'improve' in time. There is evidence to suggest that this may not be the case. The same survey questions were asked, informally, of some second-year students and some postgraduates in this department. The responses were not very different.

In traditional physics courses students are asked to think directly about the wave-particle paradox and the background of the uncertainty principle only in early years. In later courses it is assumed knowledge. Obviously further, more careful research needs to be done on this point, but our belief is that

these conceptual models are built by students in these early years and subsequently change very slowly, if at all.

The answers to both questions 3 and 4 therefore seem to be that students have constructed mental models to conceptualize the abstract concepts of quantum mechanics which have very little support from anything else in their experience. Pines and West (1983) have offered a metaphor for the social constructivist theory of learning as climbing plants growing on a trellis, and this metaphor has since been taken up by others (Roth 1990). Shoots growing upwards represent the knowledge that students construct for themselves from their own experience. The parent vine growing down towards them represents the agreed corpus of knowledge they aspire to learn. Mature learning occurs when the two intertwine. In that metaphor, the quantum mechanical mental models of the present students are slender tendrils indeed, completely unsupported by neighbours or the parent vine.

It seems to us that this interpretation of our observations might yield the answer to question 2. When asked directly why quantum mechanics is difficult most students answer something to the effect: 'It's all mathematics.' Our conclusions suggest this means that the mental models they are working with are tenuous constructs, extended far beyond the point where they are buttressed by perceived relationships with other, better understood concepts. This is probably true of many areas of university study, but it is even more so in quantum mechanics where many elements of the construct are nothing but isolated mathematical deductions balancing precariously on one another. It is little wonder that students lack confidence in performing assessment tasks required of them and hence judge the subject to be 'difficult'.

It was pointed out earlier that there is a strong tradition in physics departments for 'transmissionist' education strategies – teach them how to do the subject first, and let them worry about what it all means when they get to research level. Perhaps it is time to query that tradition, especially if, as suggested at the start of this paper, there is an increasing need for quantum mechanics to be understood by professionals who will never be researchers. This may mean not trying to answer question 1 but rephrasing it. If we change our assessment practices so that 'success' is equated with developing the kind of mental models characterized by relational richness, then our good students will pretty soon work out how to attain that kind of success.

### Notes

1. While there are many papers in the literature devoted to what topics should be taught in quantum mechanics, and how that material might be presented more palatably, there have been very few devoted to research into the educational issues involved. A representative sample of these may be found in the bibliography to Fischler and Lichtfeld (1992).
2. One of the most obvious areas of current discussion concerns Bell's Theorem. References may be found in the bibliographies of any of several recent articles, for example Mermin (1994).
3. Note that the word 'model' may have several meanings, particularly in an educational context where it is common parlance to speak of 'models of learning'. In this paper, however, we use it exclusively in the sense outlined here, namely as a 'mental model' which implies a specific set of properties.

4. It is universally agreed in textbooks on this subject that experiments in the early 1920s which demonstrated that electrons exhibited the phenomena of *diffraction and interference* clinched the debate that atomic particles could be regarded as waves.
5. In earlier years, informal tests have been given to third-year quantum mechanics classes, which included questions like this one. Whenever the answer 'They show interference and diffraction' (or something similar) was included as a multiple-choice item, between 70% and 80% of the class ticked that item. Yet here, when their memories were not prodded, half the class did not offer the standard answer as data relevant to their response.
6. The *Fourier theorem* is an important result in the classical theory of oscillations which says that any periodically repetitive function of time can be analysed into a set of harmonic oscillations. It finds many applications in the wave theory.
7. The so called *de Broglie hypothesis*, which says that any quantum particle may be considered to have a wavelength associated with it, which is inversely proportional to its momentum, was one of the great conceptual breakthroughs of the early 1920s.

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### Appendix A

It was remarked at the start of this paper that little previous work has been done to identify the key conceptual difficulties in learning quantum mechanics. One of the authors (IDJ) has been teaching this subject and has been setting short quizzes for students for some time, in an *ad hoc* kind of way. Initially these targeted the typical 'back of the chapter' problems, but the unsatisfactory nature of the responses indicated that it was the really basic ideas which students were weak on. Succeeding quizzes started concentrating on more and more fundamental concepts.

For example, the questions administered in 1993 which corresponded to those described in Section 3 of this paper had the following form.

- (1) We are all familiar with the experiments which suggest that light is made up of waves. Nowadays we say there is also evidence that light must also behave like particles. What kind of evidence are we talking about?
- (A) Light travels in straight lines and casts sharp shadows.  
 (B) A light ray can be measured to have velocity, energy and momentum.  
 (C) The energy and momentum of a light ray are proportional to one another.  
 (D) Light sometimes does *not* show effects like interference and diffraction.
- (2) We are all familiar with the experiments which suggest that electrons and protons and so on are particles. Nowadays we say there is also evidence that they must also behave like waves. What does it mean 'to behave like a wave'?
- (A) Even in the absence of forces, electrons and protons do not necessarily travel in straight lines.  
 (B) Electrons and protons are really disturbances in some underlying medium.  
 (C) Electrons and protons sometimes show effects like interference and diffraction.  
 (D) When electrons and protons collide with atoms they scatter in all directions.

...

In both of these questions there are only two responses which would be considered 'correct'. Students responses by no means unerringly lighted on these. In order to discern why those who made inappropriate choices did so, the questions given in the survey which this paper describes were simplified and made open-ended.

### Appendix B

The following are considered to be 'ideal' answers to the questions asked. Each has been assigned a marking scheme; out of 6 marks, the distribution of marks represents the relative importance of the various components of the answer (in the opinion of the two physicists involved).

#### *Question 1: What is a particle?*

A particle is an entity which has some or all of the following:

- |   | <i>Marks</i> |
|---|--------------|
| (1) It has a number of measurable properties: |              |
| it has a well-defined mass;                   | 1.0          |
| it is usually narrowly localized in space, or | 1.5          |
| it travels in a well-defined path; and        |              |
| it may have a size and some structure.        | 0.5          |

- |  |     |
|--|-----|
| (2) It behaves like a classical point object, in that it can collide with walls and other particles; | 0.5 |
| in such collisions it conserves energy and momentum;   | 0.5 |
| it responds to forces in a manner described by Newton's laws.  | 2.0 |

*Question 2: What is a wave?*

- |  |              |
|--|--------------|
|  | <i>Marks</i> |
| A wave is a phenomenon by which a disturbance (energy) propagates through a medium, without long-term change to the medium, and which exhibits properties commonly associated with other things we call 'waves', i.e.: | 1.0          |
| (1) It is usually non-localized;   | 0.5          |
| (2) it will have wavelength, frequency and amplitude (unless the initial disturbance is non-periodic);   | 1.5          |
| (3) as it moves it exhibits reflection, refraction and absorption;   | 0.5          |
| (4) (most importantly in the context of quantum mechanics) it obeys the principle of superposition, – it exhibits diffraction and interference.  | 1.0          |
|  | 1.5          |

*Question 5: What is the difference between indeterminacy and uncertainty?*

- |  |              |
|--|--------------|
|  | <i>Marks</i> |
| (1) Indeterminacy  |              |
| This is an intrinsic feature of the laws of nature.  | 1.0          |
| It says you cannot predict unambiguously what will happen in any situation on a microscopic level, except in special cases.        | 1.0          |
| All you can predict is the probability with which various allowed results will occur.  | 1.0          |
| (2) Uncertainty  |              |
| This specifically concerns a <i>measurement</i> on a microscopic scale.  | 1.0          |
| Any such must show a spread of results.  | 1.0          |
| If the same measurement is repeated under exactly the same conditions, a different result (within that spread) is likely to occur. | 1.0          |