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2013 Eur. J. Phys. 34 591

(http://iopscience.iop.org/0143-0807/34/3/591)

IOP PUBLISHING EUROPEAN JOURNAL OF PHYSICS
Eur. J. Phys. **34** (2013) 591–597 doi:10.1088/0143-0807/34/3/591

Electromagnetic wave velocities: an experimental approach

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Received 12 December 2012, in final form 16 February 2013
Published 13 March 2013
Online at stacks.iop.org/EJP/34/591

Abstract
We describe experiments with coaxial transmission lines for the study of some of the velocities used to characterize the propagation of electromagnetic waves in a medium, namely phase, group and signal velocities. The experiments are suitable for undergraduates at advanced laboratory level. Their purpose is to acquaint the students with the fact that in a dispersive medium there are many possible definitions for the speed of light, and that the measurement of these different velocities is important for general understanding of wave propagation.

(Some figures may appear in colour only in the online journal)

1. Introduction

In general, the value of the velocity of light in a medium depends on how this velocity is defined [1, 2]. The most widely known velocity is that of phase, defined as $v_p = \omega/k$ for a monochromatic plane wave of angular frequency ω and wave number k. The phase velocity determines the rate at which one frequency component of the travelling signal propagates and, as is well known, there are cases in which it is greater than the speed of light in vacuum c. Another useful velocity is that of group, perhaps not as well known as phase velocity among undergraduate students. It is defined as $v_g = \mathrm{d}\omega/\mathrm{d}k$, calculated at the central wave number of a wave packet. Group velocity can be seen as the velocity of the propagation of the overall shape of the wave packet. In some cases, group velocity describes the transport of a signal through dispersive media much better than phase velocity. Perhaps because of this, many students tend to believe group velocity is the 'true' velocity of light in a medium. Much less well known as a velocity of light is signal velocity, with which the first nonzero part of the electromagnetic signal moves [3]. This is clearly hard to measure, and an operational substitute is the velocity of the propagation of some threshold level on the leading part of the wave front. There are many other meaningful definitions for the speed of light in a medium (up to eight measures are given in [2]), but we will restrict our discussion here to these three velocities: phase, group, and signal.

Transmission lines provide a good way to study the propagation of electromagnetic signals experimentally [4–7], among other techniques [8–11]. The behaviour of signal transmission lines has been investigated since the pioneering works of Maxwell, Kelvin and Heaviside [12]. In the 19th century, Kelvin proposed a diffusion model of the current in a submarine cable and applied it to the operation of the transatlantic telegraph. Some years later, Heaviside published the first papers on signal propagation in cables (see [12] and references therein). An electric circuit can be treated as a transmission line if the time it takes for a pulse to travel through it is of the same order as the pulse rise time or greater, meaning that the circuit's elements must be treated as distributed. If the pulse propagation time is much smaller than its rise time, the dimensions of the electric circuit can be neglected and the passive circuit elements can be treated as lumped. For practical purposes, the circuit should be handled as a transmission line if its length is larger than one tenth of the wavelength for sinusoidal signals crossing it. In that case, the time delay and the reflections in the interfaces of the line must be considered.

There have been proposals for experiments using coaxial transmission lines for the study of electromagnetic wave propagation. Aksornkitti *et al* [13] described a simple experiment that demonstrates the concept of dispersion of electromagnetic pulses. Lonngreen *et al* [14] suggested that pupils can be introduced to diffusion in gases using the distributed RC transmission line as a model. Alexeff [15] projected a backward-wave line of conventional inductance-capacitor design. Landt *et al* [16] explored the analogy between wave propagation in plasmas and in a particular transmission line. In this paper we describe experiments that allow one to measure the phase, group and signal (or front) velocities in coaxial lines. One of the main advantages of the proposed setup is that it is not necessary to use very long coaxial cables to carry out the measurements.

2. Experimental setup and results

2.1. Phase velocity

Maintaining its shape while crossing a dispersive medium is an exclusive characteristic of a sine wave [17]. Such waves propagate with phase velocity and experiments that measure this velocity have been described before [13]. For the sake of comparison with the group and signal velocities, we reproduce them here. Briefly, the equipment consists of a wave generator, a dual trace oscilloscope, two coaxial cables, a 50 Ω terminator and two 'T' connectors. The scheme of the apparatus is shown in figure 1. Sine waves of variable frequency (up to 6 MHz) are produced by the wave generator. A 20 cm coaxial cable connects the generator output and the input on an oscilloscope via a connector 'T'. A second coaxial cable (in this case of 13.20 ± 0.01 m, but the longer the better) is connected in the second output from the 'T' and the other end of the cable is connected on channel 2 of the oscilloscope. The oscilloscope, like any good voltmeter, has high input impedance (∼1 MΩ) and the coaxial cable used (RG58) has an impedance of 50 Ω. A very important detail in this experiment is that, in order to avoid reflections from the channel, a 50 Ω terminator must be inserted at the end of the coaxial cable, matching the impedance of the cable with the oscilloscope. This is equivalent to putting a 50 Ω impedance in parallel with the input impedance of the oscilloscope, so that the incident wave 'sees' an effective impedance of 50 Ω. The time it takes for a wave crest to travel through the longer coaxial table is measured directly at the oscilloscope, by comparing the signals at the two channels. We obtained (68 ± 2) ns for a 2 MHz signal. From this result and the cable length we get $v_p = (0.65 \pm 0.02)c$ for the phase velocity on the coaxial cable at hand, the uncertainty being essentially due to the time measurement. This value is virtually independent of frequency in the range 2–6 MHz.

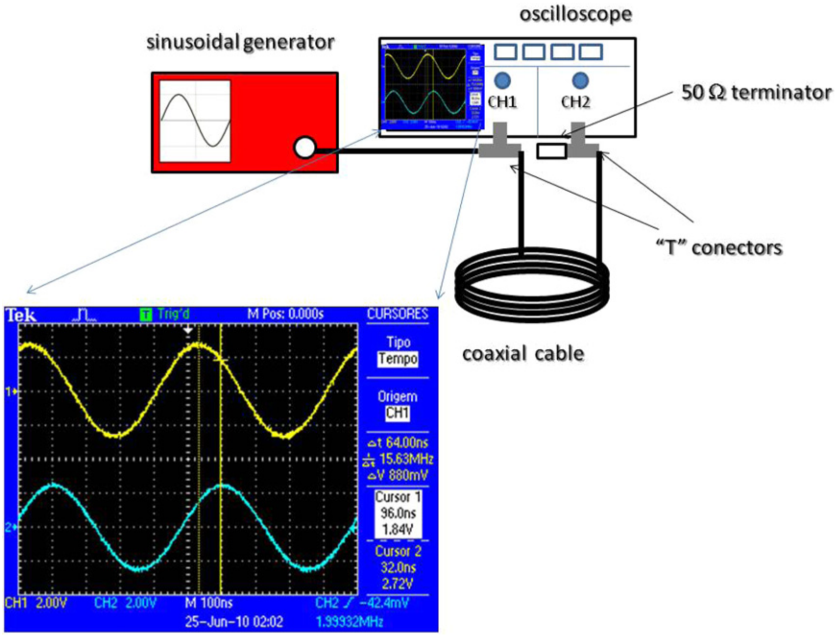

Figure 1. Schematic view of apparatus to measure the phase velocity of a sinusoidal wave in a coaxial cable.

2.2. Group and signal velocity

If the phase velocity is frequency independent, as we found in coaxial cables at the MHz range, then phase, group and signal velocities are all the same. This changes markedly at higher frequencies, as we show next by measuring the group and signal velocities of nanosecond pulses. The apparatus consists of a pulse generator (model ORTEC 480), a constant fraction (CF) discriminator (model ORTEC 584), coaxial cables, and an oscilloscope, as shown in figure 2. The pulse generator produces a slow (rise time of the order of microseconds) negative pulse. In order to obtain a fast pulse, the signal from the pulse generator is fed to the entrance of the CF, which outputs a signal with rise time of approximately 2 ns. The maximum frequency in this pulse can be estimated using the rule of thumb $f_{\max}(\text{MHz}) = 350/\text{risetime (ns)}$, which gives $f_{\max} \approx 175$ MHz. The fast negative output signal travels through a (1.99 $\pm$ 0.01) m coaxial cable to the oscilloscope. Part of the pulse is reflected back and forth (figure 3) at the interfaces between the coaxial cable (impedance $Z = 50\ \Omega$) and oscilloscope ($Z \approx 1\ \text{M}\Omega$), and coaxial cable and CF output.

For the measurement of group velocity, we adopt the maxima of the observed signals as reference points. The group velocity is then measured by time of flight, with displacements given by even multiples of the cable length. Figure 4 shows a graph of the displacement versus time for the propagation of pulses seen in figure 3. The measured group velocity is $v_g = (0.574 \pm 0.003)\ c$.

Group velocity cannot always be associated with the propagation of information. The information in a signal is transmitted with the speed of the wave front. To measure such a speed one would need a detector with infinite sensitivity. Because of this, operationally we define the signal speed as that at which a given signal level at the wave front propagates. It is necessary to select this threshold somewhat arbitrarily, and figure 5 shows the result for 10% of the signal maximum. The resulting signal speed is $v_{10\%} = (0.591 \pm 0.003)\ c$.

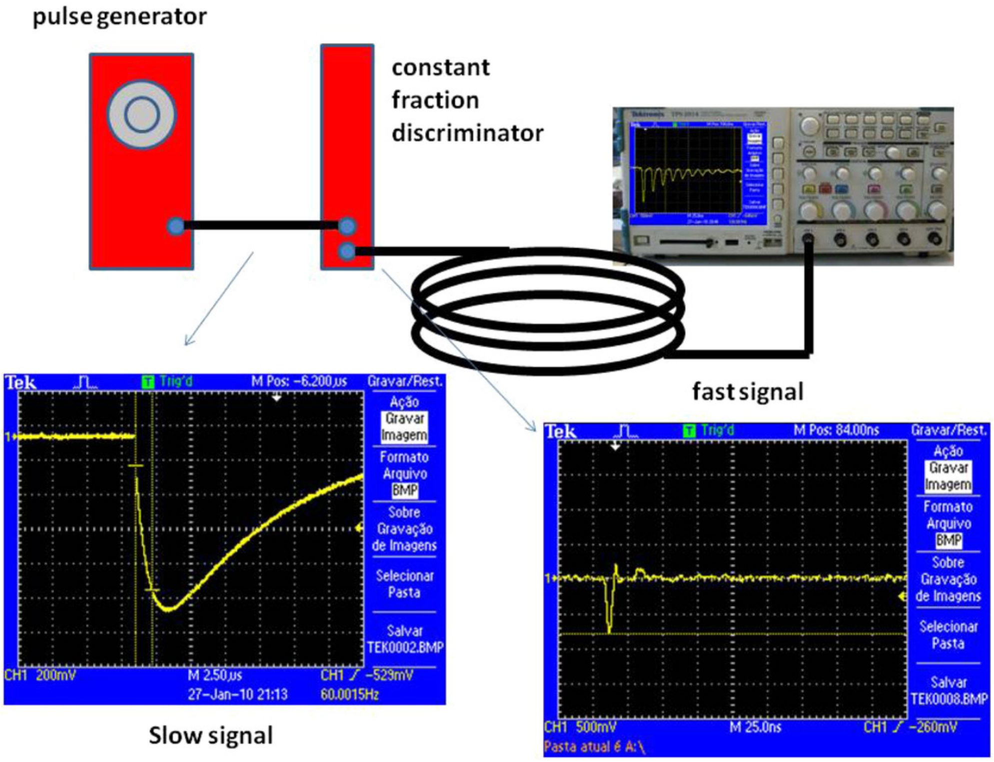

Figure 2. Schematic view of the apparatus (see text for details).

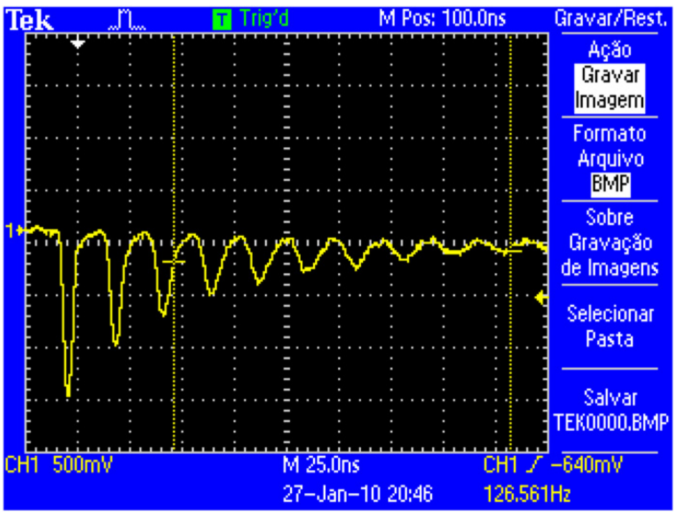

Figure 3. Multiple reflections of a pulse in a coaxial cable.

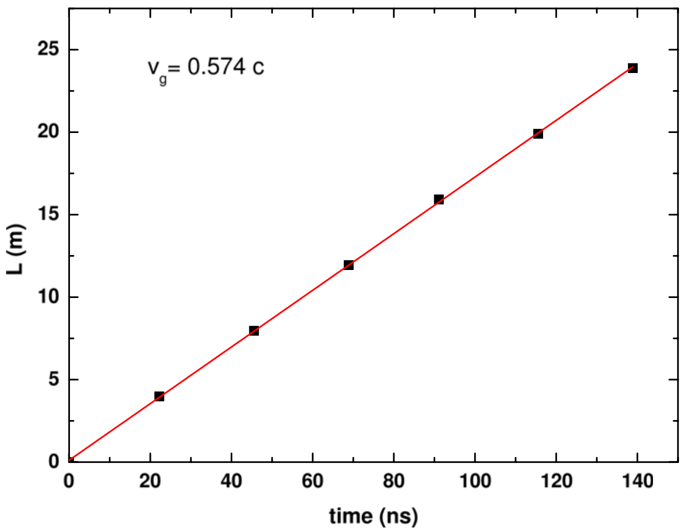

Figure 4. Distance travelled by the pulse as a function of time in a coaxial cable.

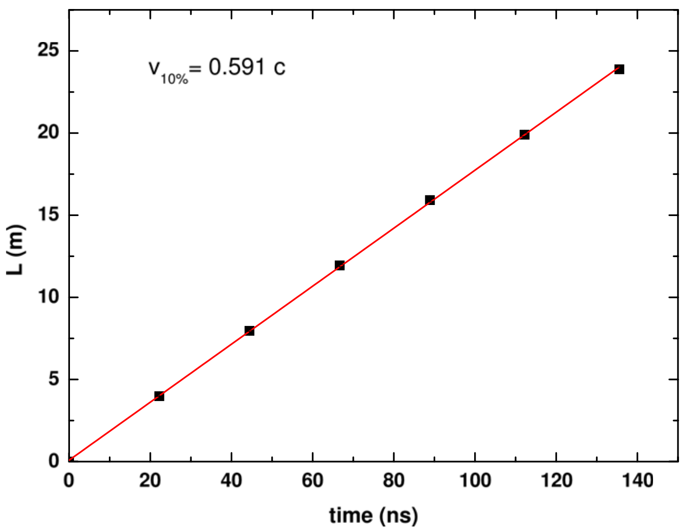

Figure 5. Position of the signal front at 10% of maximum level as a function of time.

2.3. Dispersion relation

The difference between v_g and $v_{10\%}$ indicates that the width of the pulse increases as it propagates along the cable. This is illustrated in figure 6, which shows the square of the full width at half maximum (FWHM) for the signals seen in figure 3, as a function of the square of distance L along the cable. For a Gaussian wave packet the FWHM is approximately given by [13]

$$\Delta(L) = \Delta_0\sqrt{1 + \left(\frac{8\ln 2}{(\Delta_0 v_g)^2}\frac{\mathrm{d}v_g}{\mathrm{d}\omega}L\right)^2},$$

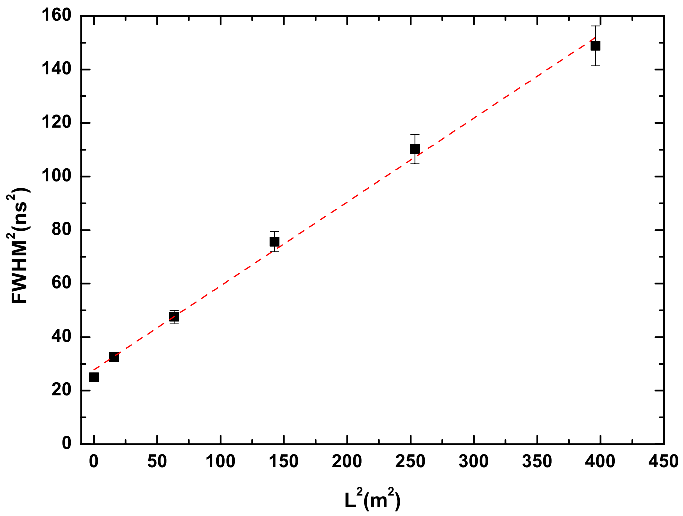

Figure 6. FWHM squared of the pulse as a function of distance squared in a coaxial cable.

where Δ_0 is the FWHM at $L = 0$. By fitting a linear function to the data shown in figure 6, one finds $\Delta_0 = (5.3 \pm 0.2)$ ns and $|\mathrm{d}v_g/\mathrm{d}\omega| = (1.6 \pm 0.1) \times 10^{-2}$ m rad^{-1}.

At high frequencies, the dispersion relation of electromagnetic waves propagating in a coaxial cable is given approximately by [18, 19]

$$k = \frac{\omega}{v_0}(1 + 2A/\sqrt{\omega}),$$

where $v_0 = 0.667\ c$, A is a constant and the frequency range is such that $A/\sqrt{\omega} \ll 1$. We can check that this dispersion relation is consistent with our results by comparing the group velocity $v_g \approx v_0(1 - A/\sqrt{\omega})$ and its derivative $\mathrm{d}v_g/\mathrm{d}\omega \approx v_0 A/(2\omega^{3/2})$ with the measured values. Taking 150 MHz as a typical frequency (consistent with the maximum value $f_{\max} =$ 175 MHz estimated from the fast pulse rise time), we obtain from the measured value of $\mathrm{d}v_g/\mathrm{d}\omega$ that the dispersion parameter is $A = 4.6 \times 10^3$ (rad s^{-1})$^{1/2}$. With this parameter we can compute the group velocity from the dispersion relation. At 150 MHz we obtain $v_g =$ $0.566\ c$, not far from the measured value, the difference being smaller than 2%.

3. Conclusions

We have used coaxial transmission lines to measure a few of the many speeds that characterize the propagation of electromagnetic waves in dispersive media as well as the dispersion relation $\mathrm{d}v_g/\mathrm{d}\omega$. Although the phase velocity was found to be virtually independent of the frequency in the 2–6 MHz range, for fast pulses containing frequencies of the order of 100 MHz we have seen that dispersion is an important phenomenon. This should help students understand that in dispersive media the speed of light has many characterizations, which differ not only in magnitude but also conceptually.

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